On the Second Order Topological Sensitivity Analysis

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Abstract: *The topological sensitivity analysis gives the topological asymptotic expansion of a cost functional with respect to an infinitesimal domain perturbation. However, for practical applications, it is necessary to consider perturbations of finite size [1].*
In this work, we have obtained an explicitly formula for the topological asymptotic expansion considering first and second order approximations. In particular, we have applied the Topological-Shape Sensitivity Method [2] in the total potential energy associated to the Laplace equation in two-dimensional domain, which was perturbed through the insertion of a small inclusion.

Topological Sensitivity Analysis for the Poisson's Problem

The variational formulation: find $u_\varepsilon \in U_\varepsilon$, such that $\int_{\Omega_\varepsilon \cup B_\varepsilon} k_\delta \nabla u_\varepsilon \cdot \nabla \eta_\varepsilon + \int_{\Gamma_\varepsilon} \bar{q}\,\eta = 0 \qquad \forall\, \eta \in V_\varepsilon$, where U_ε is the admissible functions set and V_ε is the admissible variations space, given by $U_\varepsilon = \{u_\varepsilon \in H^2(\Omega_\varepsilon \cup B_\varepsilon) : u_\varepsilon|_{\Gamma_D} = \bar{u}\}$

$V_\varepsilon = \{\eta_\varepsilon \in H^2(\Omega_\varepsilon \cup B_\varepsilon) : \eta|_{\Gamma_D} = 0\}$, and Γ_D and Γ_N are the Dirichlet and Neumann boundaries, such that $\partial\Omega = \Gamma_D \cup \Gamma_N$, with $\Gamma_D \cap \Gamma_N = \emptyset$, $\bar{u}$ and $\bar{q}$ are the temperature and heat flux prescribed on Γ_D and Γ_N , respectively.

k_δ is defined, as $k_\delta = \begin{cases} k, & \forall x \in \Omega_\varepsilon \\ \delta k & \forall x \in B_\varepsilon \end{cases}$.

The cost functional is:

$$\psi(\Omega_\varepsilon \cup B_\varepsilon) = \frac{1}{2}\int_{\Omega_\varepsilon \cup B_\varepsilon} k_\delta |\nabla u_\varepsilon|^2 - \int_{\Gamma_\varepsilon} \bar{q} u_\varepsilon .$$

The Topological-Shape Sensitivity Method gives:

$$\psi(\Omega_\varepsilon \cup B_\varepsilon) = \psi(\Omega_\varepsilon) - \pi\varepsilon^2 k \frac{1-\delta}{1+\delta} |\nabla u(\hat{x})|^2$$
$$+ \pi\varepsilon^4 \frac{k}{2}\frac{1-\delta}{1+\delta} \det \nabla\nabla u(\hat{x}) + O(\varepsilon^5).$$

where function u is solution of estate equation for $\varepsilon = 0$.

Then, we have presented some numerical experiments and observed that the estimate considering the second order topological derivative remains precise even for very large inclusions or holes, can be observed in fig. 1:

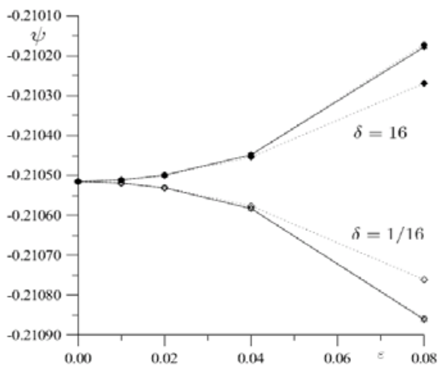

Figure 1: estimate of $\psi(\Omega_\varepsilon \cup B_\varepsilon)$.

$\delta > 1$	$\delta < 1$
$\psi(\Omega_\varepsilon \cup B_\varepsilon)$	$\psi(\Omega_\varepsilon \cup B_\varepsilon)$
$\psi(\Omega) + f_1(\varepsilon) D_T\psi$	$\psi(\Omega) + f_1(\varepsilon) D_T\psi$
$\psi(\Omega) + f_1(\varepsilon) D_T\psi + f_2(\varepsilon) D_T^2\psi$	$\psi(\Omega) + f_1(\varepsilon) D_T\psi + f_2(\varepsilon) D_T^2\psi$

References

[1] J. Rocha de Faria, A.A. Novotny, R.A. Feijóo, E. Taroco, C. Padra. Second Order Topological Sensitivity Analysis. *Int. J. of Solids and Structures*. 2007.

[2] A.A. Novotny, R.A. Feijóo, C. Padra & E. Taroco. Topological Sensitivity Analysis. *Comput. Methods Appl. Mech. Engrg*. 2003.