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Market volatility modeling for short time window

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ABSTRACT

The gain or loss of an investment can be defined by the movement of the market. This movement can be estimated by the difference between the magnitudes of two stock prices in distinct periods and this difference can be used to calculate the volatility of the markets. The volatility characterizes the sensitivity of a market change in the world economy. Traditionally, the probability density function (pdf) of the movement of the markets is analyzed by using power laws. The contributions of this work is two-fold: (i) an analysis of the volatility dynamic of the world market indexes is performed by using a two-year window time data. In this case, the experiments show that the pdf of the volatility is better fitted by exponential function than power laws, in all range of pdf; (ii) after that, we investigate a relationship between the volatility of the markets and the coefficient of the exponential function based on the Maxwell–Boltzmann ideal gas theory. The results show an inverse relationship between the volatility and the coefficient of the exponential function. This information can be used, for example, to predict the future behavior of the markets or to cluster the markets in order to analyze economic patterns.

1. Introduction

Recent advances on the understanding of the economic phenomena and its statistical properties using concepts and methods of physics have attracted the interest of researchers of different areas, such as physicists, mathematicians and economists. The market fluctuation (volatility) analysis is very important to model the dynamic of the markets and is also relevant for practical applications such as risk estimation and portfolio optimization [1].

The volatility [2] is a statistical measure of the deviation or dispersion of returns for a given share or market index. Its value can be estimated by using some deviation measure, like mean of differences, variance or standard deviation between returns from that same share or market index. Normally, the higher the volatility, the riskier the share. There are several forms of volatility [2–5], for example historical volatility, implied volatility or more sophisticated models, such as the exponentially weighted moving average (EWMA) used by RiskMetrics and GARCH process [6]. In particular, the GARCH process is introduced in order to model long-range autocorrelations in absolute returns, as found in empirical data [7], and to generate power-law tails in the distribution of returns [8]. Besides in autocorrelations, long-memory property has been found in cross-correlations between different financial variables [9]. Here, the volatility is estimated by the mean of the absolute differences between returns [4,10],

$$V_T(t) = \frac{1}{n} \sum_{t'=t}^{t+n-1} |\ln S(t' + \Delta t) - \ln S(t')|, \tag{1}$$

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over a time window $T = n \cdot \Delta t$, where n is a positive integer and Δt is the time-lag (in this work, for all data, $\Delta t = 1$ day). This equation represents the average of the n data values and can be referred to as the sample average for the scale n. If n is small, the average is more sensitive to high frequency data. If n is large, the average is more sensitive to low frequency data. This scale n can variate from 1 to the maximum number of time series observations (here, n has the $1/day$ unit).

In the literature, many approaches for market modeling have been developed observing the return series (volatility fluctuations) of the financial markets. Bachelier [11] proposed the Brownian motion to model the stochastic process of the return. Based on the central limit theorem, this approach concluded that the return over a time scale Δt follows a Gaussian Distribution. Other works developed by Mandelbrot [12,13], Fama [14] and Mantegna and Stanley [15] have claimed that the distributions of the returns can be approximated by a symmetric Lévy stable law.

Other studies also analyzed the returns of the series. For example, Cont et al. [16] proposed the use of exponentially truncated stable distributions, Eberlein et al. [17] have considered normal inverse Gaussian with asymptotical decay as power law multiplied by an exponential and Longin [18] studied the distribution of minima and maxima with a Fréchet distribution. Laherrere and Sornette [19] suggested to fit the distributions of stock returns by the Stretched Exponential (SE) law. Recently, Queiros et al. [20] proposed the fitting of financial data with a q-Gaussian distribution. Podobnik et al. [21,22] analyzed approximately 8000 stocks comprising Nasdaq and New York Stock Exchange (annually recorded). In these works the pdf of the aggregated returns are approximated by Laplace distributions (double exponential) in the broad central region.

Among several approaches to model the dynamics of the volatility fluctuations of the financial markets, the probability density function (pdf) [21,22,16] is rather important. The pdf is a function used to represent the probability distribution of a determined variable. In Econophysics, this analysis represents the relationship between high volatilities and low volatilities as the probability of occurrence of each one. This information can be used to understand economic phenomena of markets and provide new insights about the economic fluctuations, extreme events as crashes and high valorization in the value of indexes of markets, temporal evolution of money, etc [1,23,5,24,25,2].

The focus of this paper is to understand the dynamics of the world market indexes by studying all range of volatilities, using the probability density function of the series in a short window time. The idea is to find an approach that fits better to data adhering to all regions of data.

Matia et al. [3] shown that the pdf of the volatility for the Indian stocks is fitted by an exponential law, while the pdf of the volatility for American stocks is fitted by a power law in the distribution tail suggesting the existence of two classes of markets. In this way, the motivation of this work is to indicate the best approach to model the world index markets for all distribution range, thus a comparison is made with classical Econophysics approaches [1,26,27,3,28,29]: exponential function and power laws.

Eq. (2) describes the power laws, where a and k are the proportionality constant and the exponent, respectively. The power laws are quite widespread in the Econophysics literature [1,5,24,25,20] and are extensively used in various analysis, including scaling properties, relation between large and small volatilities and observation of extreme events.

$$y = a \cdot x^k. \tag{2}$$

The exponential function can be seen in Eq. (3), where the coefficients a and B are constants: a is the initial amplitude ($x = 0$) and B is a decay rate. Dragulescu and Yakovenko [27,30,31] used the exponential function [32–34,19] to analyze the distribution of money between the agents of a system. The study concluded that the distribution of money between rich and poor agents follows the Maxwell–Gibbs Distribution. This distribution is governed by an exponential function similar to

$$P(x) = a \cdot e^{-B \cdot x}. \tag{3}$$

Herein we use 17 world market indexes, from developed and developing economies, in order to evaluate exponential function and power laws in the volatility analyses task.

This paper is organized as follows. Section 2 describes the methodology of the proposed analysis. Evidences and a possible analogy between the dynamics of particle movements of an ideal gas, described by the Maxwell–Boltzmann Distribution, and the dynamics of the stock markets is presented in Section 3. Finally, Section 4 discusses the final remarks.

2. Methodology

To test the approaches, exponential function and power laws, for a short window time observation, the daily returns of seventeen world market indexes were analyzed for the period between January 2008 and January 2010. The chosen indexes are from both developed and developing markets. The elected indexes are: United States of America (S&P500 and Dow Jones Industrial Average), England (FTSE 100), Japan (Nikkei 225), Germany (Dax 30), French (CAC 40), Canada (GSPTSE), Spain (Ibex 35), South Korea (Kospi), Italy (MIB), Sweden (OMX), Norway (OSEAX) and Singapore (STI), Argentine (Merval), Mexico (IPC), India (Bse Sensex) and China (SSEC). Table 1 shows the indexes. In order to compare the performance of the exponential function against the power laws in the adjustment of all the points of the probability density function, we design the following steps:

Table 1
Fitting errors (MSE) in semi-log and log-log scales.

Indexes (countries)	Volatilities of indexes	Least squares		Trust region		Levenberg–Marquardt		Coefficient B		
		Log–log	Semi-log	Log–log	Semi-log	Log–log	Semi-log	B (LS)	B (TR)	B (LM)
Bse Sensex	1.8630	0.4244	0.0042	0.4244	0.0042	0.4244	0.0042	0.9448	0.9448	0.9448
OSEAX	1.8398	0.2909	0.0303	0.2909	0.0303	0.2909	0.0303	0.7874	0.7874	0.7874
SSEC	1.8074	0.3182	0.0064	0.3182	0.0064	0.3182	0.0064	0.8875	0.8875	0.8875
Merval	1.7870	0.2653	0.0182	0.2653	0.0182	0.2653	0.0182	0.8771	0.8771	0.8771
Nikkei 225	1.6716	0.2521	0.0250	0.2521	0.0250	0.2521	0.0250	0.8844	0.8844	0.8844
Dax 30	1.5733	0.2244	0.0305	0.2244	0.0305	0.2244	0.0305	0.8629	0.8629	0.8629
MIB	1.5399	0.2284	0.0392	0.2284	0.0392	0.2284	0.0392	0.8078	0.8078	0.8078
OMX	1.5234	0.2517	0.0069	0.2517	0.0069	0.2517	0.0069	0.9316	0.9316	0.9316
CAC 40	1.4942	0.1934	0.0106	0.1934	0.0106	0.1934	0.0106	0.9841	0.9841	0.9841
IBEX 35	1.4853	0.2436	0.0116	0.2436	0.0116	0.2436	0.0116	0.9559	0.9559	0.9559
S&P500	1.4807	0.2270	0.0174	0.2270	0.0174	0.2270	0.0174	0.8547	0.8547	0.8547
GSPTSE	1.4725	0.2422	0.0279	0.2422	0.0279	0.2422	0.0279	0.8274	0.8274	0.8274
Kospi	1.4054	0.2257	0.0292	0.2257	0.0292	0.2257	0.0292	0.8613	0.8613	0.8613
IPC	1.4028	0.2500	0.0123	0.2500	0.0123	0.2500	0.0123	0.8786	0.8786	0.8786
STI	1.3767	0.2462	0.0093	0.2462	0.0093	0.2462	0.0093	0.9003	0.9003	0.9003
FTSE 100	1.3609	0.2253	0.0432	0.2253	0.0432	0.2253	0.0432	0.9300	0.9300	0.9300
Djia	1.3561	0.2240	0.0427	0.2240	0.0427	0.2240	0.0427	0.9500	0.9500	0.9500

1. The return series is defined for each index as
$$g(t) = \frac{(\ln S(t + \Delta t) - \ln S(t))}{\delta}, \tag{4}$$
where $\Delta t = 1$ day, $S(t)$ is the market index value at time t, and δ is the standard deviation of $(\ln S(t + \Delta t) - \ln S(t))$. This formula [3] describes how the return series are constructed from the index series;
2. For each index, the volatility [10] described in Tables 1 and 2 was calculated by Eq. (1) with $n = N$, where N is the total number of time series observations. The historical volatility is used and it is based on the average of the deviation for the range of two years; here we used the historical volatility based on the average deviation of data on the period of two years.
3. The probability density function (pdf) of the return series is estimated.
4. The power law adjustment and the exponential function adjustment are compared based on four algorithms: Least Squares (LS) [35], Trust Region (TR) [36], Levenberg–Marquardt (LM) [37] and Maximum Likelihood Estimation (MLE) [38,39]. The Mean of Squared Errors (MSE) is used as measure to evaluate the results for the LS, TR and LM methods. For the MLE methodology, the negative log-likelihood [6] is used to evaluate the results, where lower the negative log-likelihood result better the adjustment.

At first we analyze the pdf of series (in all range) using linear function: LS algorithm in log–log and semi-log scales. In the semi-log scaling, the fitting corresponds to the adjustment by the exponential function, while in log–log scaling the fitting corresponds to the adjustment made by the power law adjustment. In the second step, the original data (pdf of the return series) was used and the fitting procedure was done based on three algorithms: TR, LM and MLE. We used exponential function and power laws to adjust the data without any transformation, and after that, a comparison was made with the linear fit.

The LM is an iterative algorithm which aims to minimize the cost function that is expressed as the sum of squares of a non-linear real-valued function. This algorithm is widely adopted in a broad spectrum of disciplines, such as Mathematics [37] and Computer Science [40]. The LM algorithm can be thought as a combination between the Gauss–Newton algorithm [41] and the gradient descent method [42]. When the current solution is far from the correct one, the algorithm behaves like a steepest descent method, guaranteeing the convergence.

The TR algorithm, also known as restricted step method, searches for the region which solves the minimization problem. In order to achieve it, the TR algorithm uses a model function (often a quadratic one). When the TR algorithm finds a model to the objective function, the region is expanded, trying to find new promising regions to solve the problem, conversely, if the adjustment is poor, the region is contracted, and the algorithm searches for other regions that can solve the problem.

The Maximum Likelihood Estimation was originally developed by Fisher in the 1920s and is used for fitting the parameters of a statistical model to the data. The method selects values of the parameters that maximize the probability (likelihood) of the sample data and produce the most likely distribution. The MLE methods are versatile and can be applied to most models and to different types of data. They provide efficient methods for quantifying uncertainty through confidence bounds.

The TR and the LM provide numerical solutions to the problem of function minimization. The LM and MLE are non-linear algorithms, while the TR is a linear search method.

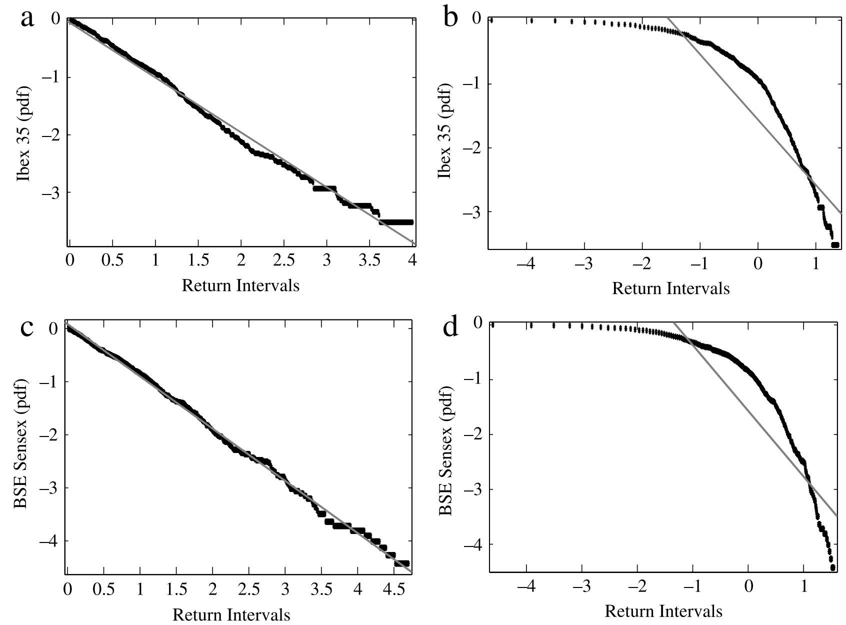


Fig. 1. Comparison of linear adjustments using LS algorithm. (a–b) Fitting to Ibex 35 index (Spain) in semi-log and log–log scales, respectively. (c–d) Fitting to BSE Sensex index (India) in semi-log and log–log scales, respectively. The black line is the probability density function (pdf) of indexes and gray line is the fit line.

Table 2
Fitting errors (MSE and likelihood) using power laws and exponential function.

Indexes (countries)	Volatilities of indexes	Trust region		MLE		Levenberg–Marquardt		Coefficient B		
		Power law	Exponential	Power law	Exponential	Power law	Exponential	B (TR)	B (LM)	B (MLE)
Bse Sensex	1.8630	0.00114	0.00011	−203.97	−513.7	0.00169	0.00011	0.9448	0.9448	0.9928
OSEAX	1.8398	0.00037	0.00031	−200.65	−541.4	0.00060	0.00031	0.9848	0.9848	0.9403
SSEC	1.8074	0.00026	0.00006	−224.19	−555.5	0.00089	0.00006	0.9070	0.9037	0.9146
Merval	1.7870	0.00058	0.00017	−312.87	−485.2	0.00107	0.00017	1.0370	1.0370	1.0499
Nikkei 225	1.6716	0.00024	0.00013	−217.01	−489.7	0.00072	0.00013	1.0580	1.0580	1.0406
Dax 30	1.5733	0.00123	0.00031	−147.77	−460.7	0.00067	0.00031	1.1280	1.1280	1.1016
MIB	1.5399	0.00121	0.00039	−175.07	−513.4	0.00067	0.00039	1.0240	1.0240	0.9934
OMX	1.5234	0.01503	0.00068	−215.17	−545.2	0.01860	0.00112	0.9892	0.9614	0.9334
CAC 40	1.4942	0.00160	0.00018	−173.07	−498.1	0.00123	0.00018	1.0690	1.0690	1.0314
IBEX 35	1.4853	0.00103	0.00013	−183.36	−515.9	0.00102	0.00013	1.0270	1.0270	0.9962
S&P500	1.4807	0.00083	0.00018	−174.47	−468.4	0.00064	0.00018	1.0720	1.0720	1.0929
GSPTSE	1.4725	0.00003	0.00001	−187.12	−501.0	0.00007	0.00001	1.0340	1.0340	1.0255
Kospi	1.4054	0.00084	0.00014	−170.61	−477.3	0.00065	0.00014	1.0820	1.0820	1.0740
IPC	1.4028	0.00047	0.00034	−235.28	−498.4	0.00101	0.00034	1.0540	1.0540	1.0308
STI	1.3767	0.00147	0.00011	−241.79	−510.7	0.00147	0.00011	0.9966	0.9966	1.0064
FTSE 100	1.3609	0.00071	0.00022	−215.51	−492.0	0.00071	0.00022	1.0660	1.0660	1.0437
Djia	1.3561	0.00061	0.00012	−215.83	−474.1	0.00061	0.00012	1.0570	1.0570	1.0807

3. Analysis

Table 1 shows the results found in first step, as described in Section 2. The results are in descending order of the volatilities. Analyzing the results, it can be seen that the best ones were obtained when the semi-log scaling was used.

Fig. 1 shows two examples of the linear fit in the semi-log and log–log scales. The black points represent the experimental observations and the gray lines are the fits. In the semi-log scale (Fig. 1a and c), the linear fit adheres better to the index probability density function because it is capable of obtaining a good adjust for all regions of the pdf of the volatility. In the log–log scale the fitting is poor because the adjustment was done for all data and not only for the tail showed in Fig. 1b and d. When the model is adjusted only for tail, the results found in the literature [1,5,24,25] show that a linear fit is a good choice.

In the second step, the exponential function and the power laws were used as described in Section 2. The results found by the three fitting methods (TR, LM and MLE) are shown in Table 2 in descending order of the volatilities. It can be seen that the exponential function obtained a better fit than the power laws in all the indexes when the TR method was used, based

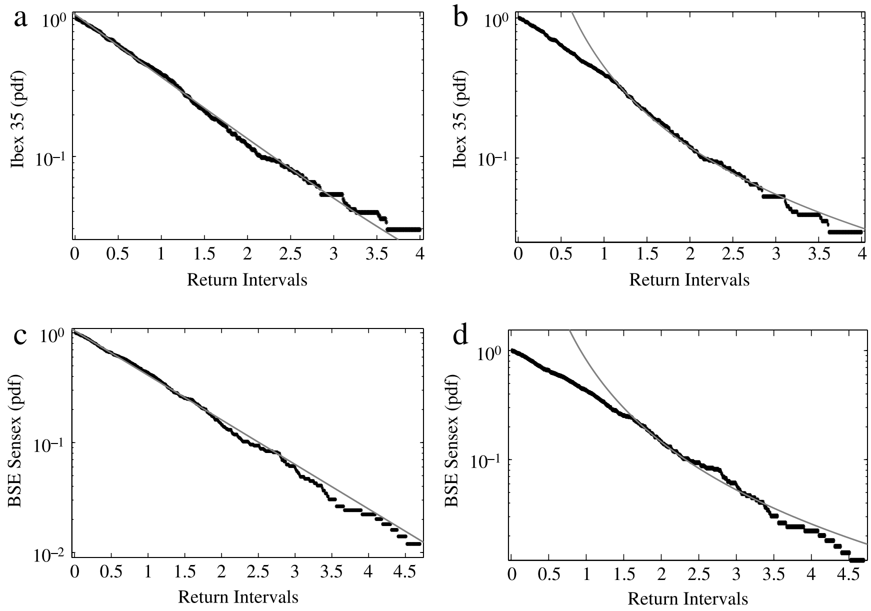


Fig. 2. Comparison of adjustments using Trust Region algorithm in semi-log scale. (a–b) Fitting to Ibex 35 index (Spain) with exponential function and power laws, respectively. (c–d) Fitting to BSE Sensex index (India) with exponential function and power laws, respectively. The black line is the probability density function (pdf) of indexes and gray line is the fit line.

on the MSE. Besides, the difference between two function adjustments is an order of magnitude in the MSE. Similarly, when the LM method was used, the exponential function was better than the power laws in the analyzed indexes. The results obtained through the Maximum Likelihood Estimation shows that all indexes are best fitted by exponential function.

Figs. 2–5 show examples of the fitting generated by the exponential function and by the power laws, where the black points are the experimental observations and the gray lines are the fits. The exponential function adheres better to the index probability density function because it is capable of doing a good adjust for all regions of the pdf, including the beginning and the end of the tail, where the pdf has a more instable behavior. However, if just the end of the tail is considered, the power law fit is very accurate and can describe this region of the pdf, as reported by Matia et al. [3] and by other works [1,5,24,25].

This instable behavior at the end of the tail occurs due to the high values in the return series. These are the cases where the investor has a higher gain or a higher loss, which can represent a possible crash [1]. Therefore, it is very important to understand the dynamics of this part of the return series, even knowing that it occurs with a low probability.

In the economic theory, there are two basic hypotheses about the stock markets: the random walk hypothesis and the non-random walk hypothesis. The random walk hypothesis is a financial theory stating that stock market evolves according to a random walk model. Therefore, the market cannot be predicted [43,44]. However, some economic scientists believe that the market is predictable to some degree (in general, weakly predictable). They believe in the non-random walk hypothesis [45,46].

If the stock markets are formed by particles (agents) and these particles do not interacting with each other (random walk hypothesis) or have a weak interaction (non-random walk hypothesis), such a system can be viewed as an ideal gas. Therefore, it is possible to suppose an analogy between the financial values of the stocks and the energy of the particles, each particle can change its energy (or velocity) with a probability that decays exponentially with the magnitude of its energy change (as observed in the pdf of the return series). This analogy, in the classical description, should follow the Maxwell–Boltzmann Distribution. Based on this assumption, it is possible to define the temperature of the market.

It is reasonable to associate the volatility with the temperature of the market. If the market is agitated its temperature is high, otherwise, if the market is not agitated its temperature is low. Temperature implies thermal energy; therefore, if the market is observed from the point of view of an ideal gas, the markets should be characterized by a given temperature (or volatility).

The Maxwell–Boltzmann Distribution [47] describes the probability function of the particle energy (or velocity), where the particles do not constantly interact with each other but move freely between short collisions (random shocks). This statistics is used to describe the distribution of the particles over various energy states in thermal equilibrium. Discarding the quantum effects, this thermal equilibrium is characterized by the low density of the particles and by the high temperature of the system. This variation in the velocity of particle generates thermal energy, resulting in an increase (or decrease) of the agitation of the particles.

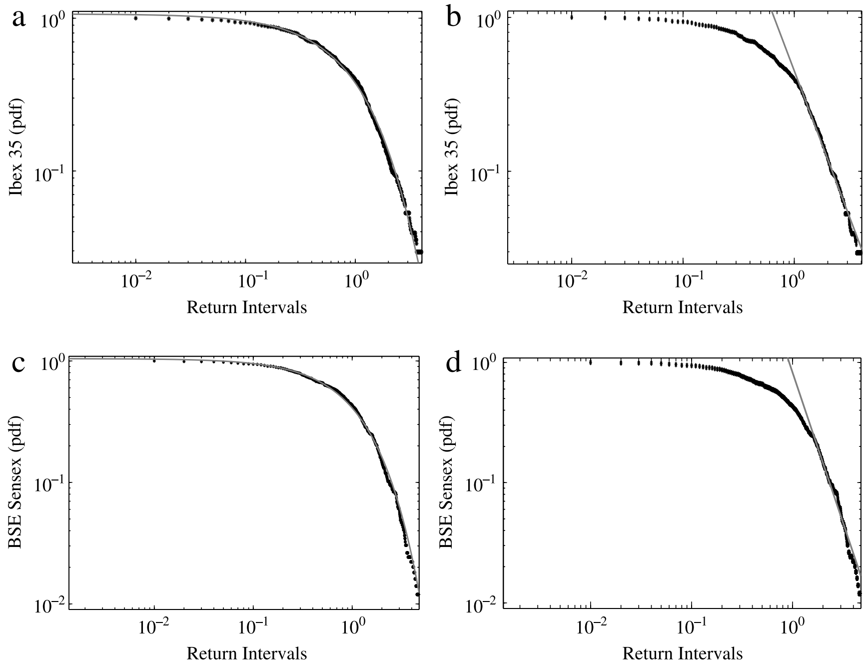


Fig. 3. Comparison of adjustments using Trust Region algorithm in log–log scale. (a–b) Fitting to Ibex 35 index (Spain) with exponential function and power laws, respectively. (c–d) Fitting to BSE Sensex index (India) with exponential function and power laws, respectively. The black line is the probability density function (pdf) of indexes and gray line is the fit line.

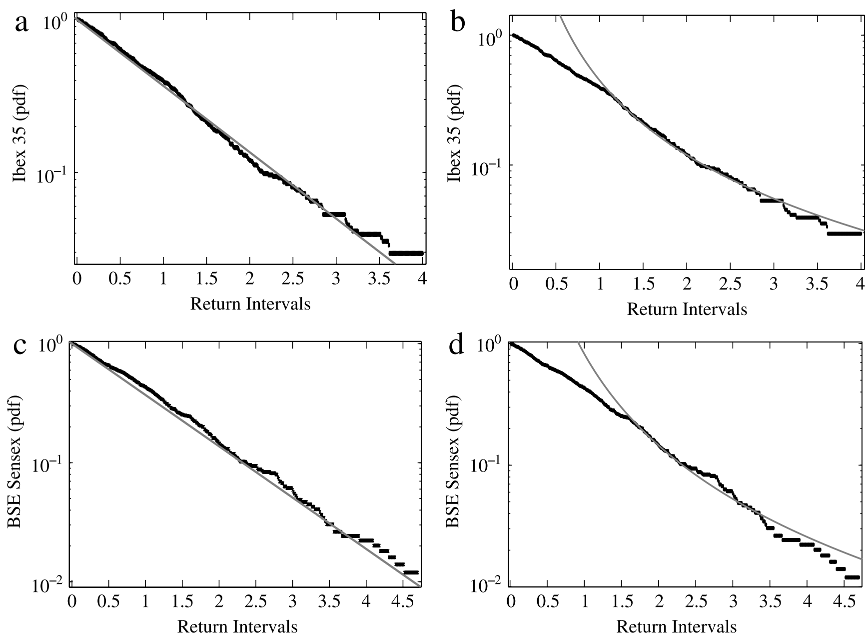


Fig. 4. Comparison of adjustments using Maximum Likelihood Estimation in semi-log scale. (a–b) Fitting to Ibex 35 index (Spain) with exponential function and power laws, respectively. (c–d) Fitting to BSE Sensex index (India) with exponential function and power laws, respectively. The black line is the probability density function (pdf) of indexes and gray line is the fit line.

Based on the context of an ideal gas, the companies' shares negotiated by investors could be compared with particles. Thus, if this assumption is plausible, the companies' shares could be described by a Maxwell–Boltzmann Distribution.

With this in mind, the temperature of an ideal gas can be related to the volatility of the market. Thus, higher temperature leads to higher volatility and lower temperature of the economic system leads to lower volatility of the market.

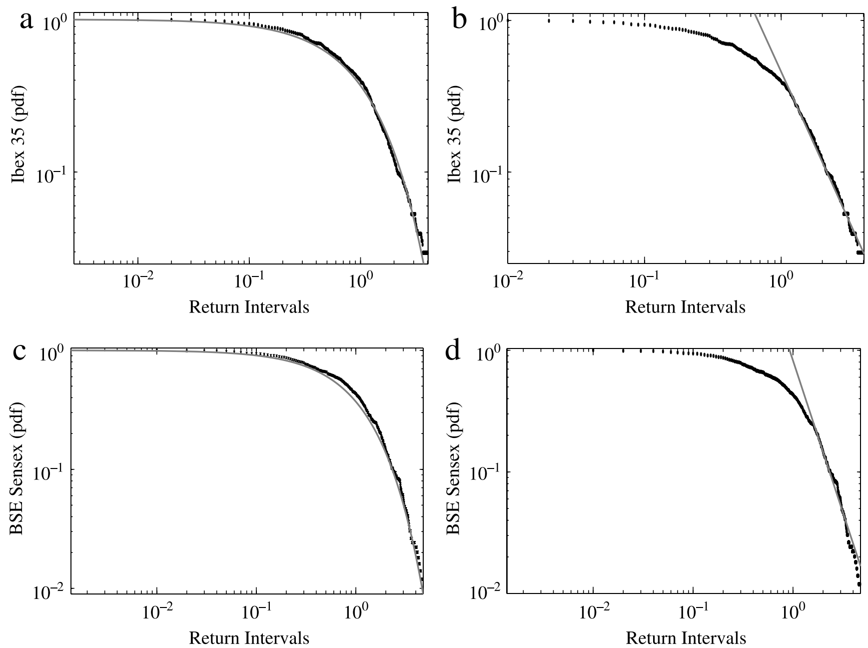


Fig. 5. Comparison of adjustments using Maximum Likelihood Estimation in log–log scale. (a–b) Fitting to Ibex 35 index (Spain) with exponential function and power laws, respectively. (c–d) Fitting to BSE Sensex index (India) with exponential function and power laws, respectively. The black line is the probability density function (pdf) of indexes and gray line is the fit line.

A relationship between Eq. (5) (the Maxwell–Boltzmann Statistics) and Eq. (3) (exponential function) is given by Eq. (6).

$$y = a \cdot \mathrm{e}^{-E/k_b \cdot t} \tag{5}$$

$$B = \frac{1}{k_b \cdot t}. \tag{6}$$

This theory corroborates with the dynamics found between the coefficients of an exponential function (Eq. (3)) and the volatility of the indexes used in this paper. The relationship found between the B coefficients of the exponential function and the system temperature (Eq. (6)) can be extrapolated to the market system, as observed in Fig. 6 and in Tables 1 and 2: when the volatility decreases (the energy decreases or the temperature decreases), the B coefficient increases. Fig. 7 shows that the relation between the B coefficient and the volatility tends to be a constant, as can be demonstrated by Eq. (7)

$$B \propto \frac{1}{\text{Volatility}} \Rightarrow B \cdot \text{Volatility} = C \tag{7}$$

where C is the proportionality constant between the B coefficient and the volatility. For the experiments with the LS algorithm, the constant found was $C = 1.38 \pm 0.08$, for the TR and LM experiments, $C = 1.53 \pm 0.07$, and for MLE experiments, $C = 1.58 \pm 0.07$.

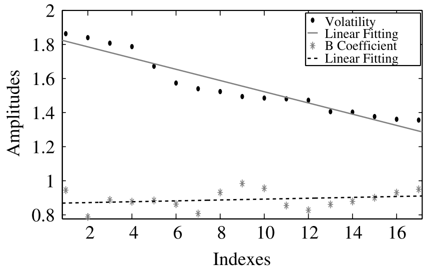
These facts indicate that the Maxwell–Boltzmann theory can be valid to analyze the pdf of the return series presented in this article. Thus, the market system could be treated like a gas system [48].

The performed experiments corroborate with the relationship between the B coefficient and the volatility given by Eq. (7). The financial risk of a given market over a specified time can be quantified by the volatility or by the B coefficient. In other words, they measure the instability/fluctuation of the markets.

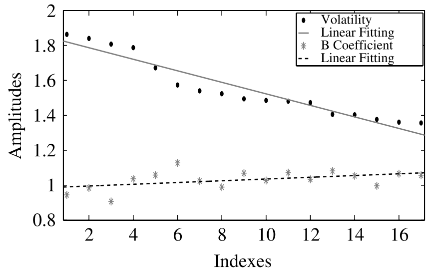
The Maxwell–Boltzmann theory describes the relationship between the particle's speeds of an ideal gas and the temperature of the system. Thus, the degree of the market's fluctuation may be supposed as the particle's energy, which is proportional to the temperature. In Eq. (6), the same relationship can be seen: the B coefficient is inversely proportional to the temperature of the system. This is a valuable information which can be used to cluster different markets or to predict the behavior of different markets.

4. Conclusions

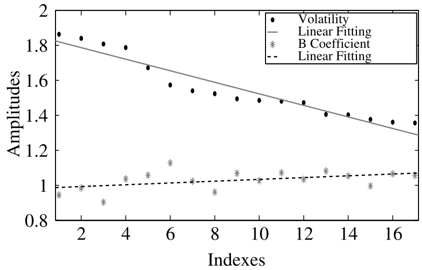
To evaluate the exponential and power law approaches, several tests were performed to find the best fitting to probability density function of the return series. The pdf describes the dynamics of the volatility fluctuations of the financial markets. The tests were performed in two steps: first, a linear fitting was done using three algorithms, Least Squares (LS) [35],



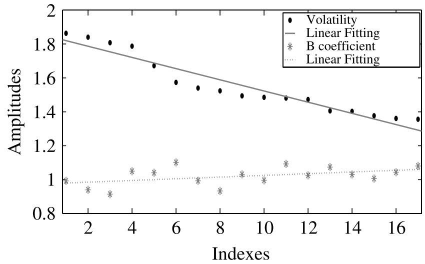
(a) Least Squares.



(b) Trust Region.

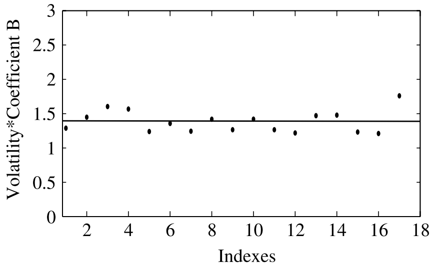


(c) Levenberg–Marquardt.

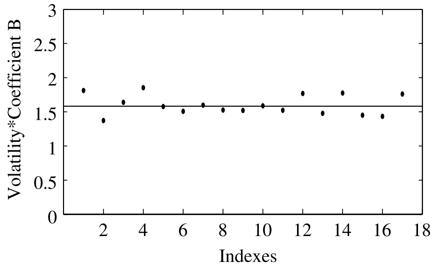


(d) Maximum Likelihood Estimation.

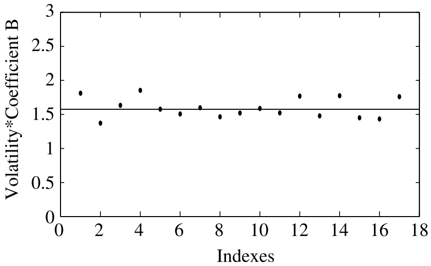
Fig. 6. Comparison between the volatility and the *B* coefficient: (a) by Least Squares, (b) by Trust Region Method, (c) by Levenberg–Marquardt method and (d) by Maximum Likelihood Estimation. In both plots, the dots are the volatilities of the indexes and the stars are the *B* coefficients of the indexes. The solid line and dashed line are the linear fitting of the volatilities and *B* coefficients, respectively.



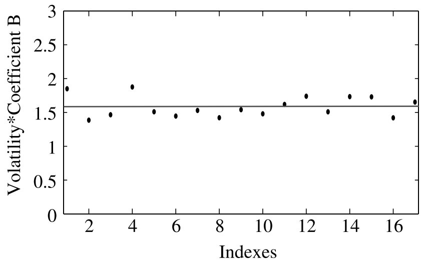
(a) Least Squares.



(b) Trust Region.



(c) Levenberg–Marquardt.



(d) Maximum Likelihood Estimation.

Fig. 7. Relationship between volatility and *B* coefficient calculated by (a) the Least Squares, (b) the Trust Region, by (c) the Levenberg–Marquardt and by (d) the Maximum Likelihood Estimation. In both plots, the dots are the product $B \cdot Volatility$ and the solid line is the mean value of these dots.

Trust Region (TR) [36] and Levenberg–Marquardt (LM) [37], after that, the fitting was performed on the original data using the exponential function and the power laws with the TR, LM and Maximum Likelihood Estimation (MLE) [38] algorithms.

The observed fitting error with the exponential function is smaller than the adjustment error obtained by the power laws for the analyzed data. The exponential function fits all regions of data while the power law fits only the tail. The exponential approach can bring insights about the modeling of the stock market volatility. This initial study demonstrates that, in the period studied, the indexes of the developing markets can be modeled by exponential function as well as the indexes of the developed markets. Therefore, this conjecture seems to be promissory to the development of a new approach to model the dynamic of the market.

The investigated series are formed by two-year observations, while the observations generally found in the literature of the Econophysics have ten or more years. Matia et al. [3] observed that developed markets (or stable markets) are guided by power laws, and developing markets are guided by an exponential function. Here, both types of markets (developed and in developing) are guided by an exponential function and the analysis is made in all range of the pdf, not only in the head or the tail. In this sense, the pdf of the volatility of the market, when analyzed in all data range, is better described by an exponential function than power laws. One possible explanation for the best performance of exponential adjust can be the short window of time used. In this case, the terms of short memory may be prevailing in relation to terms of long memory. Therefore, new experiments will be done with different time window sizes. The idea is to test other time window sizes and to analyze their behavior. Probably, there is a minimal size of the time window, that can be possible to distinguish low temperature markets from high temperature markets, through power laws and exponential function as Matia et al. [3].

Furthermore, maybe a critical length of time windows also can exist, where from this length the developed markets are described by a power law. Then, if the development markets depends on length of time window, maybe it should be possible to make a connection with critical phenomena and phase transition on markets, as studied by Sornette [49] in his theory of bubbles and crash in markets. These analyses also will be done.

Based on the Maxwell–Boltzmann Distribution, a relationship between volatility and the *B* coefficient is established for all experiments. In a finance system, the volatility can be seen as the temperature, when compared with an ideal gas system. The larger the market agitation, the higher the temperature of the markets and the higher the volatility of the index. These conclusions are supported by the results and strengthen the Maxwell–Boltzmann approach to model financial time series.

A relevant fact is the existence of a relationship between volatility and the *B* coefficient and this coefficient could be used as a new measure to quantify the movement of the markets. The *B* coefficient, according to the point of view of the market as an ideal gas system, could be used to estimate the market volatility in order to compare different markets. Normally, developed markets have a high value of *B* coefficient and developing economies have a small value of *B* coefficient. Thus, the investor can forecast the behavior of the market or can understand the relationship between high and small volatilities. This can be an interesting tool for financial market analysis.

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