Série de Fourier

Monitoria de Sinais e Sistemas

16/11/09

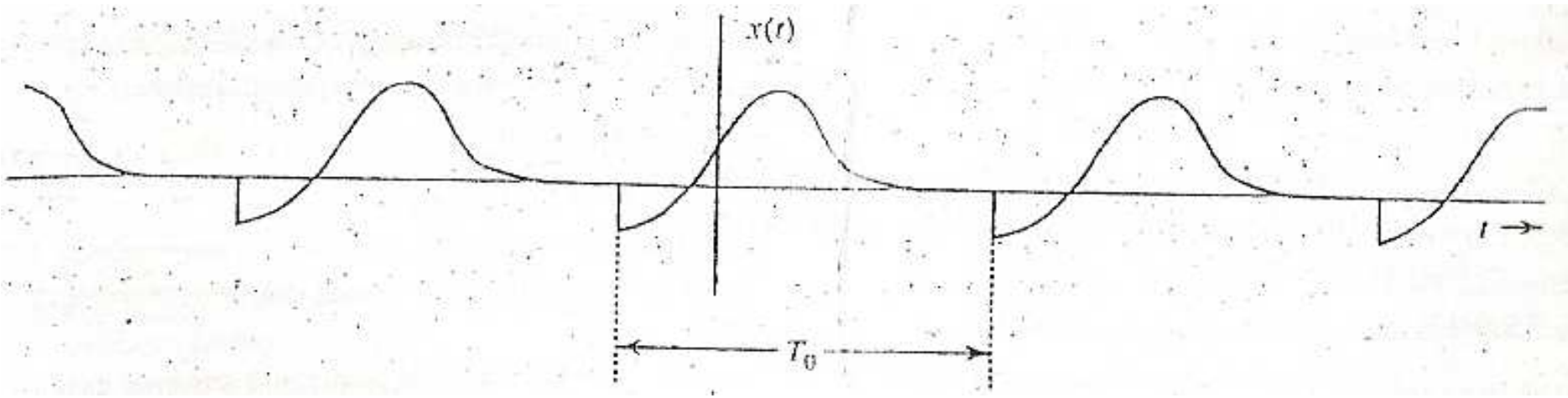
Sinais Periódicos

São Periódicos, porque...

$$x(t) = x(t + T_0) \text{ para todo t}$$

$$\int_a^{a+T_0} x(t)dt = \int_b^{b+T_0} x(t)dt$$

$$\int_{T_0} x(t)dt$$



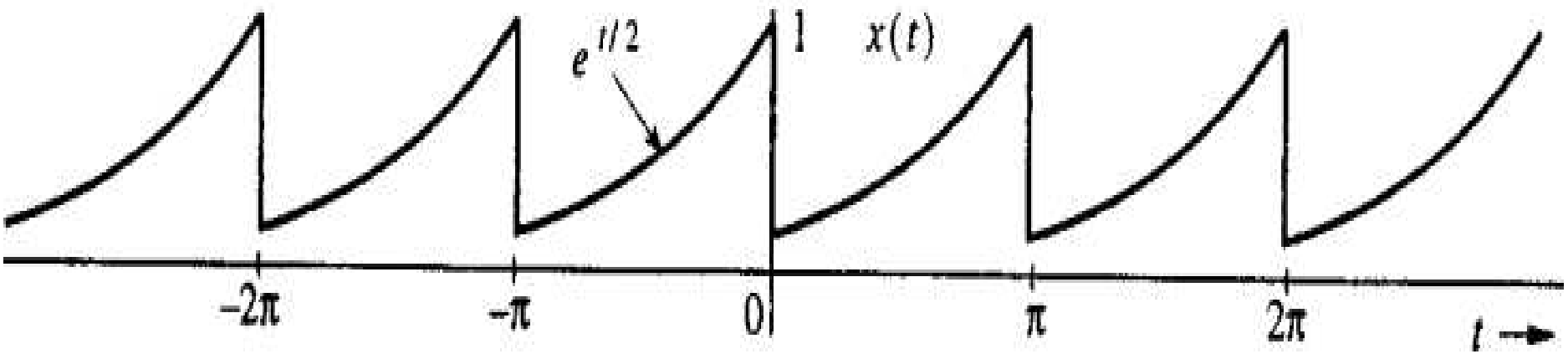
Sinais Periódicos

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nw_0 t + b_n \sin nw_0 t$$

$$a_0 = \frac{1}{t_0}\int_{t_0} x(t)dt \qquad T_0 = \frac{1}{t_0} = \frac{2\pi}{w_0}$$

$$a_n = \frac{2}{t_0}\int_{t_0} x(t)\cos nw_0 t dt \qquad b_n = \frac{2}{t_0}\int_{t_0} x(t)\sin nw_0 t dt$$

Exemplo: Exercício 6.1-2 Determine a série de Fourier para x(t) mostrada na figura:



Forma Trigonométrica Compacta

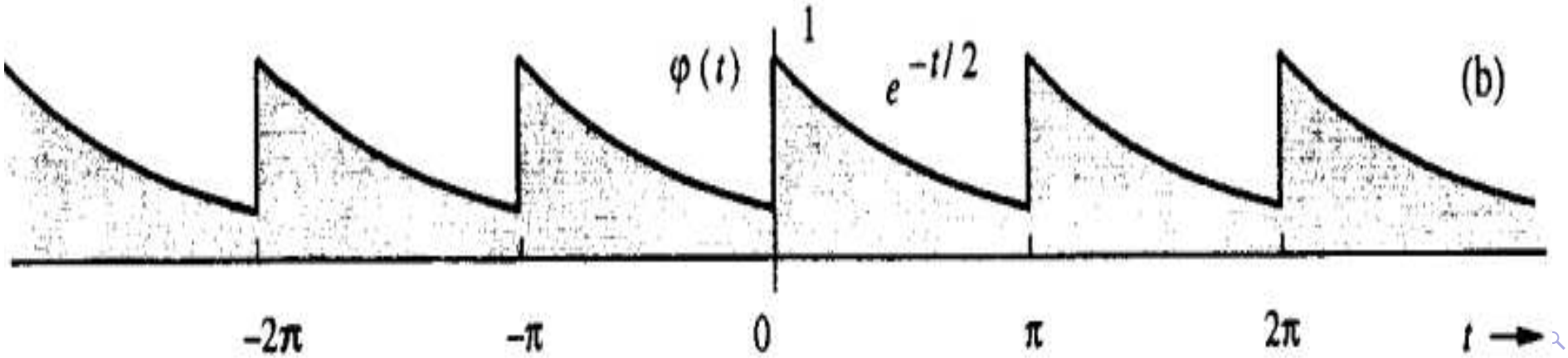
$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n w_0 t + \theta_n)$$

$$C_0 = a_0$$

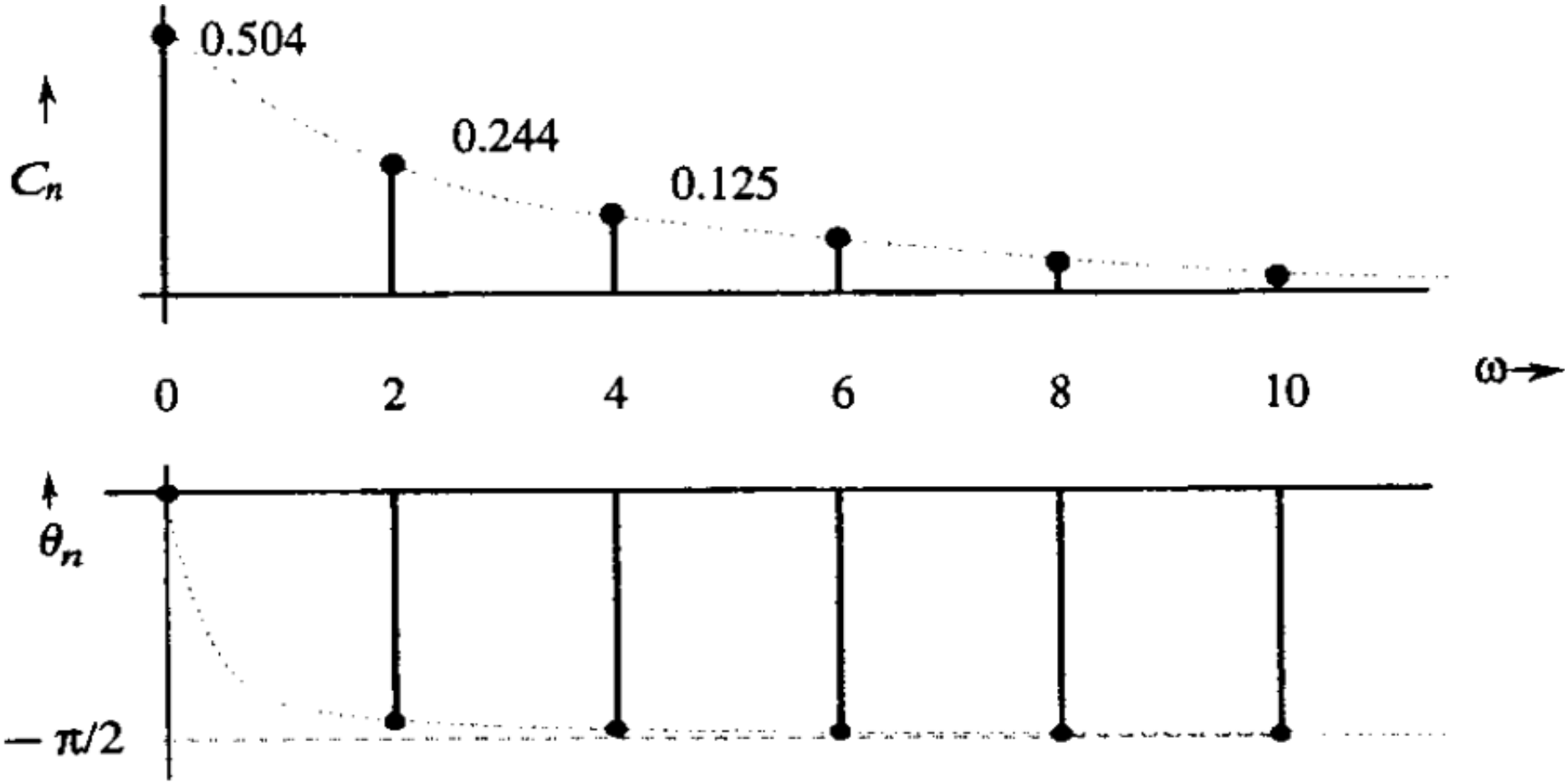
$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\Theta = \arctan\left(-\frac{b_n}{a_n}\right)$$

Exemplo: 6.1 Determine a série trigonométrica compacta de Fourier do sinal periódico x(t) mostrado na figura. Trace o espectro de amplitude de fase de x(t).



Forma Trigonométrica Compacta



Simetria

$x(t)$ é Par $\Rightarrow a_0 = \frac{2}{T_0}\int_0^{\frac{T_0}{2}} x(t)dt$

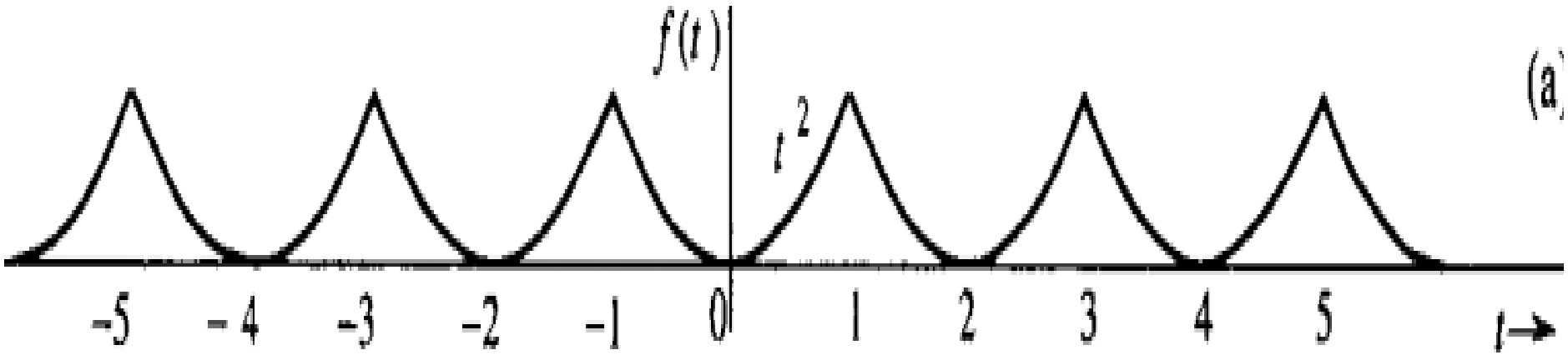
$$a_n = \frac{4}{T_0}\int_0^{\frac{T_0}{2}} x(t)\cos nw_0 t$$

$$b_n = 0$$

$x(t)$ é Impar $\Rightarrow a_0 = 0$

$$b_n = \frac{4}{T_0}\int_0^{\frac{t_0}{2}} x(t)\sin nw_0 t dt$$

Exemplo: Exercício E6.1 Determine a série trigonométrica compacta de Fourier para o sinal abaixo:



Série Exponencial de Fourier

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jnw_0 t} \qquad D_0 = a_0 = C_0$$

$$D_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jnw_0 t} dt$$

$$D_n = \frac{1}{2} C_n e^{j\theta_n} \begin{cases} |D_n| = |D_{-n}| = \frac{1}{2} C_n \\ D_n = \theta_n, \quad D_{-n} = -\theta_n \end{cases}$$

Exemplo: 6.5 Determine a série exponencial de Fourier do sinal abaixo: