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Secondary Virtual Network Mapping onto Cognitive Radio Substrate: A Collision Probability Analysis

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Abstract— This letter proposes a priority-based network virtualization mapping process based on Cognitive Radio (CR). Primary (PVNs) and Secondary (SVNs) Virtual Networks are mapped onto the same CR substrate. The SVN mapping is a NP hard problem in which not only the SVN demand must be considered, but also the PVN activities in order to ensure reduced interference level to PVN. The interactions between PVNs and SVNs are modeled by an M/M/N/N queue with priority and preemptive service. A Collision probability formulation is proposed, validated and analyzed in order to assess the SVN mapping behavior under different primary and secondary loads.

Index Terms— Cognitive Radio, Virtual Wireless Networks, Collision Probability, Secondary Virtual Networks Mapping

I. INTRODUCTION

Virtualization is put forward as a fundamental component to manage the future dense and heterogeneous wireless ecosystem, since different virtual wireless networks (VWNs) can share the same wireless infrastructure. Despite its benefits, the state of the art approaches for wireless virtualization [1] [2] provide little flexibility at the PHY and MAC layers and can cause resource underutilization (e.g. spectrum), since the allocated resources to one VWN are not shared with another during operation. As their traffic load varies over time, VWNs may not make full use of their resources, undermining the deployment of new VWNs and leading to revenue losses for the Mobile Network Operator (MNO).

The problem of resource underutilization and little flexibility may be overcome by combining wireless virtualization with the cognitive radio (CR) technology and dynamic spectrum access (DSA) techniques [3]. This cooperation enables to achieve the deepest level of wireless virtualization [4], improve resource utilization through opportunistic resource sharing and deal with different wireless technologies with no hardware modification. Thus, it is possible to have VWNs that adopt different access technologies sharing the same wireless substrate.

We envision a further step in the synergy of VWN and CR/DSA, by empowering the benefits of opportunistic sharing for wireless virtualization through VWN mapping with different access priorities onto wireless substrate. We call the higher priority virtual networks as Primary Virtual Networks (PVNs) and lower priority ones as Secondary Virtual Networks (SVNs). PVN users, denoted as Primary Users (PUs), access the wireless substrate resources allocated to its specific PVN. While SVN users (SUs) opportunistically access resources allocated to the PVNs. As the PVNs have

higher access priority, they could offer any type of application supported by wireless substrate such as voice service, multimedia and real time applications. Due to the preemption possibility of their service, the SVNs present some limitations on the types of application they support. Delay sensitive or critical time (e.g., real time applications) applications might not work as expected on this type of network. On the other hand, SVNs could offer best-effort services such as P2P download and web browsing.

Traditional VWNs mapping onto wireless substrate networks (i.e. reserving and allocating physical resources to the VWNs) is a NP-hard problem [5] and, in the current literature, it is performed without taking into account the existence of differentiated priorities [1] [2] and opportunistic resource sharing. Our mapping approach is more challenging because the SVN mapping must not only consider its own demands (e.g. number of users/requested bandwidth and QoS requirements), but also the PUs activities in order to ensure that the interference level caused to primary communication is reduced or does not go beyond a defined threshold. In this respect, the interference/collision between PUs and SUs, which occur when a PU returns to a channel that is being used by SU, must be estimated and taken into account during the SVN mapping.

To the best of our knowledge, this is the first study to define an environment composed of PVNs and SVNs that are mapped on the same wireless cognitive radio substrate; address the SVN mapping onto substrate network and to model the interactions between PVNs and SVNs by using an M/M/N/N queue with priority and preemptive service. We have also proposed, validated and analyzed a collision probability formulation, highlighting the SVN mapping behavior when different levels of primary and secondary loads are considered. The remainder of this letter is organized as follows. Section II presents the system model and a formulation for collision probability. Model validation and analysis are conducted in Section III. Section VI concludes this letter.

II. PROPOSED FORMULATION

A. System Model

We consider an environment composed of substrate network, PVNs and SVNs. The substrate networks consist of channels, spectrum bands, base stations, servers and other features that compose the infrastructure of the wireless environment and are managed by the MNO [6].

Adopting a two-level model [6], we consider that the service provider requests the creation of and manages L PVNs. The substrate network is composed of M channels that are used for virtual network mapping and a given mapping algorithm that

divides the resource between the PVNs according to a percentage q_j , with $0 \le q_j \le 1$, $\sum_{j=1}^{L} q_j = 1$, and Q_j being the set of channels allocated to PVN j, with $|Q_j| = \lfloor M. q_j \rfloor$ or $\lceil M. q_j \rceil$, where $\lceil x \rceil$ and $\lfloor x \rfloor$ are the ceil and floor functions, respectively.

We consider that the PU arrival at channel $i(C_i)$ of the virtual network j, with $C_i \in Q_j$, follows a Poisson process with arrival rate $\lambda_{PU,i,j}$, and the PU holding time is given by an exponential distribution with mean $\frac{1}{\mu_{PU,i,j}}$. Moreover, we assume each channel having capacity to satisfy one PU [7]. Given that a set of N channels was allocated to PVN $j(Q_j)$, i.e. $|Q_j| = N$, the total PU arrival rate can be obtained by Eq.1.

$$\lambda_{PU,j} = \sum_{C_i \in Q_j} \lambda_{PU,i,j} \tag{1}$$

The SVNs provide their services by using the resources allocated to the PVNs in an opportunistic manner. In this environment, there is no one-to-one relationship between PVN and SVN. Thus, channels allocated to different PVNs can be used by the same SVN, which provides more possibilities for SVN mapping. This is unlike [8], which adopts a one-toone relationship between SVNs and PVNs.

For the secondary communication, we consider Z SVNs to be mapped onto the substrate network. In each SVN l (SVN_i), with l = 1, 2, 3..., Z, the SU arrival follows a Poisson distribution with rate $\lambda_{SU,l}$ users per second. The SU holding time is given by an exponential distribution with mean $1/\mu_{SU,l}$ seconds. In a way similar to PVN, the bandwidth requested by each SU can be satisfied by one channel.

Given that the SVN_l mapping onto the substrate network adopted the set of N channels, $SC_l = \{C_1, C_2, ..., C_N\}$, where $SC_l \subset \bigcup_j Q_j$, and $SC_l \cap SC_u = \emptyset$, for all $l \neq u$, with

l, u = 1, 2, 3, ..., Z being the SVNs identifiers and that the PU service rate of the channels are homogeneous and represented as $\mu_{PU,l}$, i.e. $\mu_{PU,l} = \mu_{PU,l,l} = \mu_{PU,d,l}, \forall C_i, C_d \in SC_l$, the coexistence/interaction between PVN and SVN can be modeled as an M/M/N/N queue with preemptive-priority service, where two types of users (PU and SU) compete for *N* channels [9]. In this queueing system, resources are limited (*N* channels) and no queue (line) is allowed to be formed. Moreover, this system admits loss of the secondary user, which occurs when SU is preempted by PU and there is no available channel in the SVN_l . This user does not resume its communication at another time.

The two-dimensional state space diagram of the M/M/N/N queue is illustrated in Fig. 1. Each circle labeled *i*, *j*, with $0 \le i, j \le N$ and $0 \le i + j \le N$, represents a state where there

are *i* primary users and *j* secondary users in the system (SVN_i) . Horizontal flows to right (left) represent arrivals (departures) of PUs and vertical flows to top (down) mean arrivals (departures) of SUs. The states (i, N-i) denote a full system, where all resources are being used by PUs or SUs. Specifically, when $N-i \ge 1$, these states model situations where the SU is dropped from SVN due to PU arrival and there is no available channel to resume its communication. These states are located in the extreme right diagonal of the diagram.

B. Formulation for Collision Probability

In the SVNs mapping, it is important to consider other factors apart from the demand for these networks. As the channels adopted are shared with the PVNs, which have higher access priority, it is necessary to ensure that the interference level caused to primary communication is reduced or does not go beyond a defined threshold. Such threshold can be established on the basis of the service level agreement (SLA)/service level specification (SLS) from the PVNs or for example, the interference level that can be tolerated by the PVNs applications/signals. Thus, in selecting the channels that must be allocated to each SVN, the interference or collision probability between PU and SU must be computed to ensure that it will be reduced or lowered to the defined threshold.



Fig. 1. State transition diagram of the adopted M/M/N/N queue

A collision between PU and SU happens when a PU returns to a channel that is being used by a SU. This event damages both PU and SU communications and needs to be taken into account in the SVN mapping process. Moreover, because the PU has higher access priority to the resources, the SU has to vacate the channel and find another available channel to resume its communication. The first condition for a collision to take place is to have at least one SU in the SVN.

From the model (Fig. 1) we note that the PU arrival in the SVN_l leads to collision with SU when the SVN_l is full and

there is at least one SU in the SVN_i , i.e., for states (N - j, j), with j > 0, a PU arrival will certainly cause a collision event. Hence, the probabilities sum of these states (see Eq. 2) represents an inferior boundary to the collision probability.

$$Pc_{\inf,l} = \sum_{j=1}^{N} P(N-j,j)$$
(2)

When there is at least one SU in the SVN_i and it is not full, the PU arrival does not necessarily lead to a collision, since the PU may have returned to a channel that is not being occupied by a SU. This is modeled by states (i, j), with j > 0, and i + j < N. The sum of probabilities of these states ($\Delta col_{,i}$, in Eq. 3) denotes the probability that it might occur.

$$\Delta col_{,l} = \sum_{i=0,j=1}^{(i+j) < N} P(i,j)$$
(3)

In this situation, in order to compute the collision probability between PU and SU, it is necessary to know which channels are being used by PU, SU or which ones are not being used by both. However, the M/M/N/N model does not represent this specific information. The model expresses the steady-state probabilities in terms of how many channels are being used by PUs and SUs. It does not specify which user is using which channel. For example, given that seven channels were used to map a SVN, the steady-state probability P(2,3)(from de model M/M/7/7) only expresses the probability that there are 2 PUs and 3 SUs in the SVN. It does not indicate which channel is occupied by which user (PU or SU). Generally, this kind of information can be obtained during the network operation, because it involves channel allocation for each user individually. The SVN mapping, in turn, just deals with the allocation of a set of channels to each virtual network.

As for sates (i,0), with $i \ge 0$, the arrival of a PU does not lead to a collision, because there is no SU in the SVN_i , the collision only happens or might happen in the previous two cases. Thus, we can use Eq. 4 to estimate the collision probability in the SVN_i . It uses Eq. 2 as an inferior boundary and Eq. 3 multiplied by a factor β as an increment. The factor β aims to express how likely a collision may occur when the SVN_i is in states (i, j), with j > 0, and i + j < N.

$$Pc_{I} = Pc_{\inf I} + \beta \Delta col_{I} \tag{4}$$

One way to define β is using the average probability that the PU returns to the channel while the SU is using it, as given by Eq. 5. So, for each channel *i* allocated to SVN_i , the probability that PU returns to the channel during the SU communication $(P_{back,i})$ is calculated, i.e., the probability of the OFF time of the channel (PU is absent) being lower than the SU service time.

$$\beta = \frac{\sum_{i=1}^{N} P_{back,i}}{N}$$
(5)

Given that the PU arrival rate in a channel *i* follows a Poisson process with rate $\lambda_{PU,i}$ and that the PU service time is defined by an exponential distribution with rate $\mu_{PU,i}$, the channel's mean OFF period ($\overline{T_{OFFi}}$) is given by Eq.6, and the exponential distribution that describes the OFF times of the channel *i* has rates (λ_{OFFi}) given by the inverse of $\overline{T_{OFFi}}$.

$$\overline{T_{OFFi}} = \frac{1}{\lambda_{PU,i}} - \frac{1}{\mu_{PU,i}} \tag{6}$$

Assuming that the SU service time follows an exponential distribution with rate μ_{SU} and using λ_{OFFi} , the probability that the OFF time of the channel is lower than the SU service time is given by Eq.7.

$$P_{back,i} = P[OFF \ time < SU \ service \ time] = \frac{\lambda_{OFFi}}{\lambda_{OFFi} + \mu_{SU}}$$
(7)

Proof: Being $f_{OFFi}(y) = \lambda_{OFFi} e^{-y\lambda_{OFFi}}$, with $y \ge 0$, the probability density function (p.d.f) of the variable *Y* that describes the OFF periods of the channel *i* and $f_{SU}(x) = \mu_{SU} e^{-x\mu_{SU}}$, with $x \ge 0$, the p.d.f of the random variable *X*, which models the SU service time. Expressing a random variable *Z* in terms of *X* and *Y* as $Z = \frac{Y}{X}$, we can obtain the *P*[*OFF time < SU service time*], i.e., *P*[*Y < X*], by calculating the *P*[*Z* ≤ 1].

As X and Y are independent random variables, the joint pdf $f_{OFFiSU}(x, y)$ is given by product between the pdfs of X and Y, as shown in Eq. 8.

$$f_{OFFiSU}(x, y) = \lambda_{OFFi} \mu_{SU} e^{-\lambda_{OFFi} y} e^{-\mu_{SU} x}$$
(8)

From joint pdf, we may get the cumulative distribution function (CDF) for Z (see 9).

$$P[Z \leq z] = P\left\lfloor \frac{Y}{X} \leq z \right\rfloor = P[Y \leq zX]$$

$$P[Z \leq z] = P\left[\frac{Y}{X} \leq z\right] = P[Y \leq zX]$$

$$P[Z \leq z] = \int_{0}^{\infty} \int_{0}^{zz} \mu_{SU} e^{-\mu_{SU}x} \lambda_{OFFi} e^{-\lambda_{OFFi}y} dy dx$$

$$= \int_{0}^{\infty} \mu_{SU} e^{-\mu_{SU}x} \lambda_{OFFi} \int_{0}^{zz} e^{-\lambda_{OFFi}y} dy dx$$

$$= \int_{0}^{\infty} \mu_{SU} \int_{OFFi} e^{-\mu_{SU}x} \left[\frac{e^{-\lambda_{OFFi}y}}{-\lambda_{OFFi}}\right]_{0}^{zz} dx$$

$$= \mu_{SU} \left[\frac{e^{-\mu_{SU}x}}{-\mu_{SU}}\right]_{0}^{\infty} - \frac{e^{-(\mu_{SU} + \lambda_{OFFi}z)x}}{-(\mu_{SU} + \lambda_{OFFi}z)}\right]_{0}^{\infty}$$

$$F_{z}(z) == \frac{(\mu_{SU} + \lambda_{OFFi}z) - \mu_{SU}}{(\mu_{SU} + \lambda_{OFFi}z)} = \frac{\lambda_{OFFi}z}{(\mu_{SU} + \lambda_{OFFi}z)}$$
(9)

Taking the derivative of F_z to get the pdf of Z, we have (see Eq. 10):

$$f_{z}(z) = \frac{dF_{z}(z)}{dz} = \frac{\lambda_{OFFi}(\mu_{SU} + \lambda_{OFFi}z) - \lambda_{OFFi}(\lambda_{OFFi}z)}{\left(\mu_{SU} + \lambda_{OFFi}z\right)^{2}}$$
$$f_{z}(z) = \frac{\lambda_{OFFi}\mu_{SU}}{\left(\mu_{SU} + \lambda_{OFFi}z\right)^{2}}$$
(10)

Next, by using integral, we may get P[Y < X], as shown in Eq. 11.

$$P[Y < X] = P[Z < 1] = \int_{0}^{1} f_{z}(z) dz = \int_{0}^{1} \frac{\lambda_{OFFi} \mu_{SU}}{\left(\mu_{SU} + z \lambda_{OFFi}\right)^{2}} dz$$
$$P[Y \le X] = \frac{\lambda_{OFFi}}{\mu_{SU} + \lambda_{OFFi}}$$
(11)

III. VALIDATION MODEL AND COLLISION ANALYSIS

To validate our formulation, we have adopted a simulation approach using MATLAB software. Given a particular scenario, we compared the results obtained from the analytical and simulation models. In the simulator, the collision probability was defined as the ratio between the number of collisions and the number of PU arrivals.

We considered a scenario composed of two channels that are shared by SVN and PUs (from PVNs). The PU service rate and the SU service rate were defined as 1 and 0.1 (users per second), respectively. The PU arrival rate (in users/s) in each channel was varied (from 0.1 to 0.9) in order to analyze the collision behavior with different PU loads. In a similar way, the model was evaluated considering different SU arrival rates (ranging from 0.2 to 2.5 users/s) so as to reproduce different SU loads.

To compare the obtained results, 10 simulation instances were performed for each evaluated point (case). The simulation time was 10,000s. The average results are presented considering a 95% confidence level, which were obtained by using the Bootstrap method, with 'resample' size and number of (re) samplings equal to 10 and 1000, respectively. In Fig. 2, 'Model' and 'Sim' mean results obtained through the analytical model and simulation, respectively.

Fig. 2 presents results in terms of collision probability. Although Eq. 4 only estimates the collision probability, the results obtained by using it have behaved similarly to those from simulation. Thus, when the collision ratio tends to go down, the collision probability (from model) decreases as well and when the first increases, the latter also rises.

In addition, Fig 2 shows that, in this scenario, when the PU arrival rate increases, the collision probability/rate decreases. At first, it seems that these results do not make sense, once when the PU arrival rate increases, the PU load also increases and it is expected that the collision probability/rate would also increase. This is true when we are addressing collision in media access control situation, where users compete with each other to get access to the channel simultaneously and a higher user arrival rate leads to a higher collision possibility.

However, as shown in Section II, for a collision between PU and SU to take place it is necessary that the channel to which the PU will return is currently being used by a SU. Thus, we 4

note that SU needs access opportunities to the channel for a collision to happen. If there are less access opportunities, the collision possibility tends to reduce as well. So, in Fig. 2, when the PU arrival rate increases, implying higher PU load and fewer chances for opportunistic access, the collision probability decreases. Moreover, as the SU service time is higher (on average) than the channels OFF time (period in which the PU is not using it), when the SU gets the access to the channel, it is very likely that the SU will still be using the channel when the PU returns.

In addition, Fig. 2 shows that the collision probability increases when the SU arrival rate increases. With higher load of SUs in the SVN, it is more likely to have SUs using the channels when the PUs return.



Fig. 2. Collision probability obtained by model and simulation

IV. CONCLUSIONS AND FUTURE DIRECTIONS

In this letter, we have combined CR, DSA techniques and wireless virtualization in order to overcome the resource underutilization problem and deal with different wireless technologies with no hardware modification. In this new scenario, the SVN mapping emerges as a challenging problem, where the interference triggered to PVN must be taken into account. Hence, we have proposed and validated a collision probability formulation and analyzed its behavior in the SVN mapping under different SU and PU loads.

Future directions include modeling performance metrics such as SU blocking, SU dropping probabilities and resource utilization. In addition, a multi-objective formulation for the SVN mapping problem and the design of a scheme to solve it are envisioned as future studies.

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