Classification and Segmentation of Visual Patterns Based on Receptive and Inhibitory Fields

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Abstract

This paper presents a new model to realize a supervised image segmentation task. It is based on the concept of receptive fields that intends to analyze pieces of an image considering not only the pixels or group of them, but also the relationship between them and their neighbors, called segmentation and classification with receptive fields (SCRF). Also, in order to work with the SCRF model, is proposed here a new artificial neural network, called I-PyraNet, which is a hybrid implementation of the recently described PyraNet and the nonclassical receptive fields inhibition. Furthermore, the model and the network are applied together in order to realize a satellite image segmentation task.

1. Introduction

Image segmentation consists in the problem of partitioning a given data set in a certain number of clusters. Its aim is basically to separate the most relevant parts in an image. Most of the work developed in that way uses unsupervised segmentation to realize its job, but if the intention is to recognize objects inside the picture this kind of segmentation implies necessarily in a further analysis of the image to classify each cluster. In another work, Meurie et al. [1] showed that supervised pixel classification in an image might leads to better results when compared to the ones obtained with unsupervised techniques. A supervised segmentation task was realized in [2] where different classifiers were used to realize a skin detection job. In their results analysis, the Multilayer Perceptron (MLP) [3] presented the highest classification rate among other algorithms. However, it is important to note that the classification in [2] was realized pixel-by-pixel based on the fact that the human skin has a very consistent colors which are distinct from the colors of many other objects, but this assumption cannot be made to any given object. To solve this problem a very simple and effective model is here proposed, called *segmentation and classification with receptive fields* (*SCRF*) that makes use of the concepts of receptive fields [4] which intends to recognize an image based not only in the values of a single pixel but also in the way that it interacts with its neighborhood. What the SCRF model proposes is to divide an image in such a way that the classification for each sub-image generated is used to obtain the classification for each pixel in the image.

In another work, Phung and Bouzerdoum [5] proposed a neural network (NN), called PyraNet, to work as a supervised classifier where the input was a regular 2-D image. Its architecture allows the feature extraction and classification of a pattern in one step, making use of several advantages of 2-D NNs, like retaining the spatial topology of the image patterns while extracting features. However, this NN did not contemplate the inhibitory behavior of the neurons that surrounds any given receptive field [6]. This kind of inhibitory stimulus showed to be very common around the receptive fields and was successfully applied by [7] in the problem of contour detection achieving great results. Therefore, in this paper is also proposed a new hybrid implementation of the PyraNet, called *I-PyraNet*, based in all the concepts presented in the PyraNet and also in the inhibitory behavior around the receptive fields. The I-PyraNet is used together with the SCRF technique to realize supervised segmentation tasks.

Our contributions in this paper are threefold. First, we present the SCRF model which makes use of the pixel neighborhood to extract its classification. Next, we present the I-PyraNet classifier that is able to receive as direct inputs the sub-images generated by the SCRF model. Finally, in our experimental results are presented the advantages of integrate the SCRF model and the I-PyraNet classifier.

The paper is organized as follows. In Section 2 is pre-

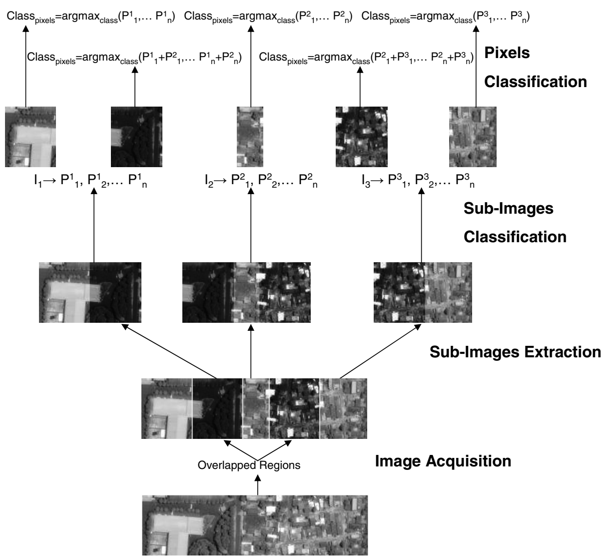

Figure 1. System architecture.

sented the model used to realize the image segmentation, the SCRF. In Section 3 is presented the I-PyraNet classifier. In Section 4 is made a comparison between different segmentation methods to solve the problem of forest detection in a satellite image. Finally, in Section 5 it is drawn some concluding remarks.

2. Image Classification Model

The model proposed in this work intends to divide an image in sub-images in order that the classification of each one of them might be used to classify each pixel in the image. It is done based on the concepts of the receptive field over an image where the basic idea is to generate sub-images sharing some overlapped pixels, leading to the advantage that the classification of a pixel will not depend only on itself, but it will depend also on the classification of the sub-images that contains it, what means that the pixels in its neighborhood will also affect its classification. Figure 1 presents the model proposed.

The model here proposed works as follows. First, an image is acquired, (Image Acquisition, Figure 1); after that the original image must be divided in sub-images (Sub-Images Extraction, Figure 1) that will have a defined size $r \times r$ called the receptive field sharing some overlapped pixels. Then, must be calculated the probability of each sub-image belongs to each one of the known classes (Sub-Images Classification, Figure 1) through the utilization of a supervised classifier. Finally, in order to classify each image pixel, after classify each sub-image, the model define the class of a pixel as the class that shows the highest sum of probabilities between the sub-images that contains it (Pixels Classification, Figure 1). However, if the pixel is not in an overlapped area, what means that only one sub-image contains it, only one probability will be generated for each class and it will be assigned to the highest one. The next equation shows how the classification is calculated for a single pixel:

$$C_{x_{i,j}} = \underset{class\ c}{\operatorname{argmax}} \left(\sum_{SI | x_{i,j} \in SI} P(c, SI)) \right), \qquad (1)$$

where $x_{i,j}$ is a single pixel in the (i, j) image position, $C_{x_{i,j}}$ is the pixel classification, c denotes one of the possible classes, SI represents a sub-image and $P(c, SI)$ represents the a posteriori probability of a given sub-image SI belongs to a given class c. Although in this work the summation of the probabilities is used to define the pixel class, any other metric using the probabilities calculated can be applied.

3. I-PyraNet

The I-PyraNet is proposed here in order to work as the supervised classifier presented in the SCRF model. Its inputs are the sub-images generated and the outputs are the classifications for each sub-image.

The original form of the I-PyraNet is the PyraNet [5] that is an artificial neural network (ANN) for classification of visual patterns, it is motivated by the very good results obtained by the convolutional neural networks (CNNs) [8]. PyraNet main advantage is that it realizes features extraction and classification into a single structure keeping the topology information of the input image. However, the PyraNet only consider the excitatory effects of neurons inside a receptive field. Therefore, is proposed here the I-PyraNet where a neuron might send excitatory or inhibitory signals to other neurons in posteriors layers based on the concept of inhibitory and receptive fields. It is important to note that the magnitude of the signal sent by a neuron will always be the same, only the direction of the signal will be affected by the fact of the neuron is present in a receptive or inhibitory field.

3.1 I-PyraNet Architecture

The architecture of a I-PyraNet is formed by a multilayered network with two different kinds of processing layers. 2-D layers, arranged into a 2-D array, that realize the feature extraction and data reduction and are located at the base of the network; and 1-D feedforward layers that realize the classification step and are located at the top of the network. The last 2-D layer is connected with the first 1-D layer. The entire network is connected in cascade, the output of one layer works as the input to the next layer. Each neuron of a 2-D layer l is connected to a receptive field in the previous

2-D layer $l-1$. Being rxr the size of this receptive field, o the amount of neurons overlapped between adjacent receptive fields and g the gap given by $g = r - o$, then the height H and the width W of the 2-D layer l is related by

$$H_l = \lfloor((H_{l-1} - o_l)/g_l)\rfloor \tag{2}$$

$$W_l = \lfloor((W_{l-1} - o_l)/g_l)\rfloor \tag{3}$$

where l denotes the index of a 2-D layer and $l = 0$ denotes the input image. Besides that, there is the vertical or horizontal amount of inhibitory neurons surrounding the receptive field of a neuron given by h, contributing negatively for the output of this neuron.

The output of a 2-D neuron consists of a nonlinear activation function f applied upon the weighted sum of the output of those neurons contained in its receptive field minus the weighted sum of the neurons contained in its inhibitory field. Then, supposing that (u, v) is the position of a neuron in the pyramidal layer l, (i, j) is the position of a neuron in the previous pyramidal layer $l-1$ and $b_{u,v}$ is the bias of the neuron (u, v), the output of the neuron y_u, v is given by

$$y_{u,v} = f\left(\sum_{i,j\in R_{u,v}} w_{i,j} * y_{i,j} - \sum_{i,j\in I_{u,v}} w_{i,j} * y_{i,j} + b_{u,v},\right), \tag{4}$$

where $w_{i,j}$ denotes the weight associate with the input position (i, j) to the 2-D layer l, $y_{i,j}$ the output of the neuron (i, j) in the previous layer $l-1$, $R_{u,v}$ the receptive field of the neuron (u, v) with the size given by r and $I_{u,v}$ the inhibitory field of the neuron (u, v) with the size given by h. The inputs to the first 2-D layer is the input image. The output of the last pyramidal layer is rearranged into a column vector and works as the input to the first 1-D layer.

1-D layer works as a simple Multilayered Perceptron (MLP), where the output of a neuron is given by a nonlinear activation function applied upon the weighted sum of the neurons connected to it and the weight is related with the connection neuron to neuron. Then, the output of the neuron y in position n at layer l, represented by y_n^l is calculated through the next equation,

$$y_n^l = f\left(\sum_{m=1}^{N_{l-1}} w_{m,n} * y_m^{l-1} + b_n^l\right), \tag{5}$$

where N_{l-1} represents the amount of neurons in the previous 1-D layer $l-1$, $w_{m,n}$ is the synaptic weight from the neuron m in the layer $l-1$ to the neuron n at layer l, y_m^{l-1} is the output of the neuron m at layer $l-1$ and b_n^l is the bias of the neuron n at layer l. The output of the last 1-D layer work as the PyraNet output. The architecture of the I-PyraNet can be seen at the Figure 2.

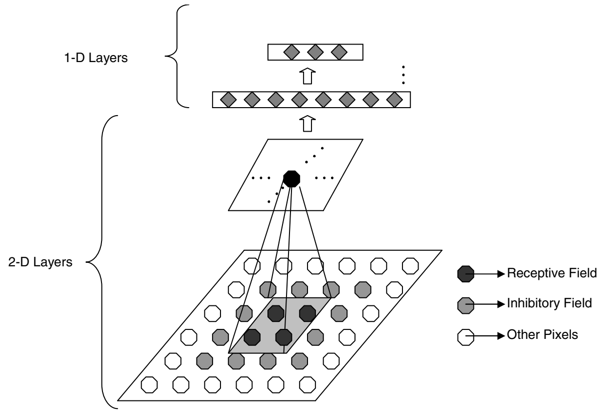

Figure 2. I-PyraNet architecture

3.2 I-PyraNet Training

In order to be able to realize the visual pattern recognition tasks, the I-PyraNet must first be trained. As a supervised neural network its objective is to reduce the error obtained through the output desired and the output obtained and it is made adjusting the synaptic weights in the I-PyraNet. The approach to realize this task used in this work is the cross-entropy function (CE) [9] where the network outputs estimate the a posteriori probability for each known class. Being y_n^L the output of the neuron n in last network layer L for an input image k, the estimated a posteriori probability p_n is given by

$$p_n^k = exp\left(y_n^{L,k}\right) / \sum_{i=1}^{N_L} exp\left(y_i^{L,k}\right), \tag{6}$$

where N_L is the amount of neurons in the layer L. Therefore, in order to adjust the synaptic weights in the I-PyraNet, the gradient error of the weights must be calculated through the error sensitivity for each neuron.

The error sensitivity δ for each neuron n at the 1-D output network layer L_{1D}, for an input image k is given by

$$\delta_n^{L_{1D},k} = e_n^k f'\left(s_n^{L_{1D},k}\right), \tag{7}$$

where e_n^k is the output y_n^k produced by the neuron n at the last 1-D layer L_{1D} minus the desired output d_n^k, then $e_n^k = y_n^k - d_n^k$. And $s_n^{L_{1D},k}$ is the weighted sum input to neuron n at layer L_{1D} and f' is the differential of the activation function f. Then, for the neurons in the others 1-D layers $l_{1D} < L_{1D}$ the error sensitivity is given by

$$\delta_n^{l_{1D},k} = f'\left(s_n^{l_{1D},k}\right) * \sum_{m=1}^{N_{l_{1D}+1}} \delta_m^{l_{1D}+1,k} * w_{n,m}, \tag{8}$$

where $N_{l_{1D}+1}$ represents the amount of neurons in the next layer $l_{1D}+1$, $w_{n,m}$ is the synaptic weight from the neuron n

in the layer l_{1D} to the neuron m at layer $l_{1D}+1$ and $\delta_m^{l_{1D}+1,k}$ is the error sensitivity of the neuron m at layer $l_{1D}+1$.

The error sensitivities for the last 2-D layer are calculated using the previous equation but rearranged into a 2-D grid. In the others 2-D layers l_{2D}, the error sensitivity for each neuron at the position (u, v) is given by

$$\delta_{u,v}^{l_{2D},k} = f'\left(s_{u,v}^{l_{2D},k}\right) * w_{u,v} * \sum_{i=i_l^{max}}^{i_h^{max}} \sum_{j=j_l^{max}}^{j_h^{max}} \gamma_{i,j}^{l_{2D}+1,k}, \quad (9)$$

where $s_{u,v}^{l_{2D},k}$ is the weighted sum input for the neuron (u, v), $w_{u,v}$ is the weight associate to the neuron (u, v) at layer l_{2D} and $\gamma_{i,j}^{l_{2D}+1,k}$ is given by

$$\gamma_{i,j}^{l_{2D}+1,k} = \begin{cases} \delta_{i,j}^{l_{2D}+1,k} & i_l \le i \le i_h, j_l \le j \le j_h \\ -\delta_{i,j}^{l_{2D}+1,k} & otherwise \end{cases}, \quad (10)$$

being $\delta_{i,j}^{l_{2D}+1,k}$ the error sensitivity for the neuron (i, j) at the next layer, and i_l^{max}, i_h^{max}, j_l^{max}, j_h^{max}, i_l, i_h, j_l and j_h are calculated by

$$i_l^{max} = \left\lceil \frac{u - (r_{l+1} + h_{l+1})}{g_{l+1} - h_{l+1}} \right\rceil + 1, i_l = \left\lceil \frac{u - r_{l+1}}{g_{l+1}} \right\rceil + 1 \quad (11)$$

$$i_h^{max} = \left\lfloor \frac{u - 1}{g_{l+1} - h_{l+1}} \right\rfloor + 1, i_h = \left\lfloor \frac{u - 1}{g_{l+1}} \right\rfloor + 1 \quad (12)$$

$$j_l^{max} = \left\lceil \frac{v - (r_{l+1} + h_{l+1})}{g_{l+1} - h_{l+1}} \right\rceil + 1, j_l = \left\lceil \frac{v - r_{l+1}}{g_{l+1}} \right\rceil + 1 \quad (13)$$

$$j_h^{max} = \left\lfloor \frac{v - 1}{g_{l+1} - h_{l+1}} \right\rfloor + 1, j_h = \left\lfloor \frac{v - 1}{g_{l+1}} \right\rfloor + 1 \quad (14)$$

Therefore, the error gradient to the weights and the biases can be obtained through the next equations.

- 1-D Weights: the error gradients for the 1-D synaptic weight $w_{m,n}$ from the neuron m at layer $l_{1D}-1$ to the neuron n at layer l_{1D} for all the images input K, are given by

$$\frac{\partial E}{\partial w_{m,n}} = \sum_{k=1}^{K} \delta_n^k y_m^{l_{1D}-1,k}. \quad (15)$$

- 2-D Weights: the 2-D synaptic weight $w_{u,v}$ of neuron (u, v) at layer l_{2D} to layer $l_{2D}+1$ is calculated by

$$\frac{\partial E}{\partial w_{u,v}} = \sum_{k=1}^{K} \left\{ y_{u,v}^{l_{2D},k} \times \sum_{i=i_l^{max}}^{i_h^{max}} \sum_{j=j_l^{max}}^{j_h^{max}} \gamma_{u,v}^{l_{2D}+1,k} \right\}, \quad (16)$$

- Biases: the error gradients for the bias of neuron n, b_n, at 1-D layer l_{1D} and of neuron u, v, $b_{u,v}$, at 2-D layer l_{2D} are respectively given by

$$\frac{\partial E}{\partial b_n} = \sum_{k=1}^{K} \delta_n^k, \frac{\partial E}{\partial b_{u,v}} = \sum_{k=1}^{K} \delta_{u,v}^k \quad (17)$$

The weights in this work are then recalculated through the use of the Gradient Descent [10] and this completes the training phase of the I-PyraNet.

4. Satellite Images Segmentation

The task realized here intends to detect forested areas in graylevel satellite images through a supervised approach. The model proposed must be compared with other existing algorithms of segmentation. Several methods of unsupervised segmentation used in many other situations are used here in comparison to the results of the SCRF model with the I-PyraNet classifier. Those methods are k-Means [11], Otsu [12] and Fuzzy C-Means (FCM) [13]. Also, two supervised techniques are used here. The k-NN pixel-by-pixel [1] applied upon graylevel pixels; and the Multilayer Perceptron [2] which is the only classifier applied upon colored pixels. The SCRF model is applied using a k-NN classifier with statics measures of the pixels inside the sub-images (mean, standard-deviation, kurtosis and skewness), too.

All the satellite images used in this work were taken from the map service Google Maps™ [14] and are 900×450 pixels, they can be accessed in the Web[1]. Two images taken from Manaus, a Brazilian city, representing forested and not forested area are used as the training images to all the supervised classifiers. The test images were extracted from three cities of different regions from Brazil (Jundiai, Manaus and Recife) and were manually segmented for comparison purposes. The classification rate is calculated by a comparison made pixel-by-pixel, where the error rate of a given image is the amount of the pixels misclassified divided by the total number of pixels in the image.

The SCRF model used in this work generates sub-images of size 18×18 with an overlapping of 6 pixels, generating an amount of 1250 sub-images. First, it is analyzed the results of the combination between the SCRF and the k-NN classifier for different values of k, presented in Figure 3. It is easy to see that the best classification result happened with $k = 100$ that will be the value for k used to classify the other satellite images.

The I-PyraNet utilized in this work has two pyramidal layers with a receptive field of size 4×4 and 5×5 for the first and the second pyramidal layer, respectively, and

[1] http://cin.ufpe.br/~bjtf/SCRF

Table 1. Error Rate in % of Forest Detection

Algorithm:	SCRF-IPN	SCRF-PN	SCRF-NN	PN	100-NN	MLP	K-Means	Otsu	FCM
Jundiai-1	**10.39**	13.81	28.17	17.11	37.81	16.87	23.64	23.41	23.41
Jundiai-2	**10.17**	12.00	19.80	13.86	35.77	28.96	35.16	36.01	33.07
Jundiai-3	**8.04**	9.34	13.61	10.91	35.77	30.15	22.53	21.99	21.14
Manaus-1	6.51	6.19	**5.67**	7.04	55.73	23.75	13.97	13.79	13.17
Manaus-2	6.26	6.24	**5.49**	6.67	53.63	27.3	22.97	23.27	21.48
Manaus-3	8.83	8.51	**6.48**	8.98	16.03	8.98	45.52	45.69	46.90
Manaus-4	**8.88**	10.13	14.79	11.50	31.75	34.52	36.67	36.67	34.98
Recife-1	2.97	**2.85**	3.65	3.34	48.26	23.98	15.98	16.14	15.35
Recife-2	**3.09**	3.43	4.91	4.20	47.18	23.18	16.95	17.38	16.66
$\bar{x}$	**7.24**	8.06	11.40	9.29	40.21	24.19	25.93	26.04	25.13

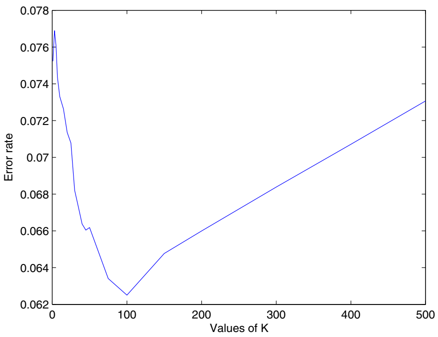

Figure 3. Error rate, vertical axis, for different values of k, horizontal axis, in the combination between SCRF and the k-NN classifier.

an overlap of 2 pixels for both layers. The network output has two neurons each one giving the a posteriori probability of the input image belong to a forested or to a not-forested region. In order to analyze the best inhibitory parameters for the I-PyraNet combined with the SCRF model, Table 2 presents the error rate obtained to classify all the test images with different sizes of the inhibitory field, where the first line shows the size of the inhibitory field for the first and the second layers of the I-PyraNet, respectively. The inhibitory fields with sizes 2 and 1 presented the best results. Then, this configuration will be used to compare with the other classifiers. It is important to see that a PyraNet is nothing more than an I-PyraNet with all the values of the inhibitory fields equals to 0.

Finally, in Table 1 is presented the average classification error rates for all the algorithms, where SCRF-IPN, SCRF-PN and SCRF-NN represent the combination of the SCRF model with the I-PyraNet, PyraNet and the k-NN classifier respectively. PN is the application of the PyraNet classifier alone, what means that it is applied like a sliding window

Table 2. Error Rate with different Inhibitory Fields Sizes

Inhibitory Field	0,0	1,0	1,1	2,1	2,2
Error Rate	8.06	12.06	8.26	**7.24**	8.85

through the entire image. The 100-NN represents the results obtained with the k-NN pixel-by-pixel classifier with $k = 100$ and the MLP is the Multilayer Perceptron classifier with color inputs. There are presented the error rate for all the test images and the means among them all. First, it is easy to see that the k-NN pixel-by-pixel and the unsupervised algorithms had the highest error rate among all the algorithms, the MLP classifier had a smaller improvement in the results, reaching an amount of 24.19% in the error rate. In other hand, the application of the SCRF model together with the PyraNet reduced the error rate in 1.23% in comparison to the classifier that uses only the PyraNet. Also, the application of the I-PyraNet reduce the error rate even more, reaching the lowest error rate of 7.24%.

Also, in order to perform a more detailed comparison between the used algorithms, Table 3 shows the computational time required to classify one image of 900×450 pixels in seconds.

The time spent with the k-NN classifiers are the highest, and the 100-NN realizing a comparison pixel-by-pixel takes seven hours to classify a single satellite image. The MLP classifier also present a slow classification time, it might be explained because the classification is made pixel-by-pixel. The unsupervised algorithms demonstrated to be faster when compared to those ones, but if this comparison is made with the other supervised algorithms they are much slower with an exception to the Otsu method that is the fastest among all the algorithms. It is important to see that the use of the SCRF model decreases a lot the classifica-

Table 3. Classification Speed (Seconds)

Algorithm	Time Spent
SCRF-IPN	0.58
SCRF-PN	0.56
SCRF-NN	77.25
PN	0.47
100-NN	25200.00
MLP	12.73
K-Means	2.05
Otsu	0.18
FCM	5.60

tion time in comparison with the pixel-by-pixel approaches, while the use of inhibitory fields increased the classification time of the regular PyraNet in only 0.02 seconds. Then, it is easy to see that the SCRF model and the I-PyraNet classifier brought gains in the classification rate without prejudicing the time spent in the recognition task.

5. Conclusions

In this work is proposed a new model for realize the segmentation and classification of images. The basic idea behind the model is to consider not only the value of the pixel itself, but it also looks at its neighbors to define the pixel class. Also, was proposed a hybrid implementation of the PyraNet based on the idea of inhibitory fields, called I-PyraNet.

The results obtained with the SCRF showed that it is a very good model to be applied with any of the two classifiers, k-NN and I-PyraNet, the last one presented the best results among all the classifiers. Although, only these two classifiers have been used in the SCRF model, any other supervised classifier can be applied instead. We have applied the methods proposed here in the classification of satellite images taken from the map service Google Maps™ [14]. In some cases it achieves an error of only 3% in the amount of wrong pixels when compared to manually segmented images. In general the errors presented were not higher than 10%. Then, it is easy to see that the results obtained with the methods here proposed had better results than the other ones.

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