Teoria Axiomática dos Conjuntos, 2009.1
Lista de Exercícios 6

Exercício 1 (4.1 (Cap. 4, Sec. 4) (5,0)) Assume that $(A_1, <_1)$ is similar to $(B_1, \prec_1)$ and $(A_2, <_2)$ is similar to $(B_2, \prec_2)$. Prove that:
(a) The sum of $(A_1, <_1)$ and $(A_2, <_2)$ is similar to the sum of $(B_1, \prec_1)$ and $(B_2, \prec_2)$, assuming that $A_1 \cap A_2 = \emptyset = B_1 \cap B_2$.
(b) The lexicographic product of $(A_1, <_1)$ and $(A_2, <_2)$ is similar to the lexicographic product of $(B_1, \prec_1)$ and $(B_2, \prec_2)$.

Exercício 2 (4.5 (Chapt. 4, Sect 4) (20 points)) Let $\langle (A_i \ <_i) \mid i \in I \rangle$ be an indexed system of mutually disjoint linearly ordered sets, $I \subseteq \mathbb{N}$. The relation $\prec$ on $\bigcup_{i \in I} A_i$ defined by: $a \prec b$ if and only if either $a, b \in A_i$ and $a <_i b$ for some $i \in I$ or $a \in A_i$, $b \in A_j$ and $i < j$ (in the usual ordering of natural numbers) is a linear ordering. Prove that if all $<_i$ are well-orderings, so is $\prec$.

Exercício 3 (5.4 (Chapt. 4, Sect 5) (20 points)) Show that a dense linearly ordered set $(P, <)$ is complete if and only if every nonempty $S \subseteq P$ bounded from below has an infimum.

Exercício 4 (5.5 (Chapt. 4, Sect 5) (20 points)) Let D be dense in $(P, <)$, and let E be dense in $(D, <)$. Show that E is dense in $(P, <)$.

Exercício 5 (6.2 (Chapt. 4, Sect 6) (20 points)) Show that $|\mathbb{N}^{\mathbb{N}}| = 2^{\aleph_0}$. [*Hint*: $2^{\mathbb{N}} \subseteq \mathbb{N}^{\mathbb{N}} \subseteq \mathcal{P}(\mathbb{N} \times \mathbb{N})$.]