161, Winter, 2005–6
Homework 7, Due Mar 10

Exercise 1 (1.4 (Chapt. 5, Sect 1) (25 points)) Prove that $\kappa^\kappa \leq 2^{\kappa\cdot\kappa}$.

Exercise 2 (1.8 (Chapt. 4, Sect 1) (25 points)) Let X be any set and let f be a one-to-one mapping of X into itself such that $f[X] \subset X$. Prove that X must be infinite.

Exercise 3 (2.3 (Chapt. 5, Sect 2) (25 points)) Show that if a linearly ordered set P has a countable dense subset, then $|P| \leq 2^{\aleph_0}$.

Exercise 4 (2.5 (Chapt. 5, Sect 2) (25 points)) Show that, for $n > 0$, $n \cdot 2^{2^{\aleph_0}} = \aleph_0 \cdot 2^{2^{\aleph_0}} = 2^{\aleph_0} \cdot 2^{2^{\aleph_0}} = 2^{2^{\aleph_0}} \cdot 2^{2^{\aleph_0}} = (2^{2^{\aleph_0}})^n = (2^{2^{\aleph_0}})^{\aleph_0} = (2^{2^{\aleph_0}})^{2^{\aleph_0}} = 2^{2^{\aleph_0}}$.