161, Winter, 2005–6
Homework 5, Due Feb 22

Exercise 1 (1.2 (Chapt. 4) (15 points)) Prove:
(a) If $|A| < |B|$ and $|B| \leq |C|$, then $|A| < |C|$.
(b) If $|A| \leq |B|$ and $|B| < |C|$, then $|A| < |C|$.

Exercise 2 (2.2 (Chapt. 4) (15 points)) Prove that if X and Y are finite, then $X \times Y$ is finite, and $|X \times Y| = |X| \cdot |Y|$.

Exercise 3 (2.4 (Chapt. 4) (15 points)) Prove that if X and Y are finite, then X^Y has $|X|^{|Y|}$ elements.

Exercise 4 (3.1 (Chapt. 4) (15 points)) Let $|A_1| = |A_2|$, $|B_1| = |B_2|$. Prove:
(a) If $A_1 \cap A_2 = \emptyset$, $B_1 \cap B_2 = \emptyset$, then $|A_1 \cup A_2| = |B_1 \cup B_2|$.
(b) $|A_1 \times A_2| = |B_1 \times B_2|$.
(c) $|\mathrm{Seq}(A_1)| = |\mathrm{Seq}(A_2)|$.

Exercise 5 (3.5 (Chapt. 3) (15 points)) Let A be countable. The set $[A]^n = \{S \subseteq A \mid |S| = n\}$ is countable for all $n \in \mathbb{N}$, $n \neq 0$.

Exercise 6 (3.6 (Chapt. 3) (25 points)) A sequence $\langle s_n \rangle_{n=0}^{\infty}$ of natural numbers is *eventually constant* if there is $n_0 \in \mathbb{N}$, $s \in \mathbb{N}$ sucht that $s_n = s$ for all $n \geq n_0$. Show that the set of eventually constant sequences of natural numbers is countable.