161, Winter, 2005–6
Homework 2, Due Jan 25

Exercise 1 (3.2 (Chapt. 2) (15 points)) The functions $f_i : i = 1, 2, 3$ are defined as follows:

$$f_1 = \langle 2x - 1 \mid x \text{ real}\rangle,$$
$$f_2 = \langle \sqrt{x} \mid x > 0\rangle,$$
$$f_3 = \langle 1/x \mid x \text{ real}, x \neq 0\rangle.$$

Describe each of the following functions, and determine their domains and ranges: $f_2 \circ f_1$, $f_1 \circ f_2$, $f_3 \circ f_1$, $f_1 \circ f_3$.

Exercise 2 (3.3 (Chapt. 2) (15 points)) Prove that the functions f_1, f_2, f_3 from the previous exercise are one-to-one, and find the inverse functions. In each case, verify that $\operatorname{dom} f_i = \operatorname{ran}(f^{-1})$, $\operatorname{ran} f_i = \operatorname{dom}(f_i^{-1})$.

Exercise 3 (3.6 (Chapt. 2) (30 points)) Prove:

(a) If f is a function, $f^{-1}[A \cap B] = f^{-1}[A] \cap f^{-1}[B]$.

(b) If f is a function, $f^{-1}[A - B] = f^{-1}[A] - f^{-1}[B]$.

Exercise 4 (3.7 (Chapt. 2) (10 points)) Given an example of a function f and a set A such that $f \cap A^2 \neq f \restriction A$.

Exercise 5 (3.9 (Chapt. 2) (30 points))

(a) Show that the set B^A exists. [*Hint:* $B^A \subseteq \mathcal{P}(A \times B)$.]

(b) Let $\langle S_i \mid i \in I\rangle$ be an indexed system of sets; show that $\prod_{i \in I} S_i$ exists. [*Hint:* $\prod_{i \in I} S_i \subseteq \mathcal{P}(I \times \bigcup_{i \in I} S_i)$.]