161, Winter, 2005–6
Homework 1, Due Wed 18

Exercise 1 (20 points) Suppose S is a set. Let $T = \{x \in S \mid x \notin x\}$. Prove that T exists and is unique. Show that T is *not* an element of S. [*Note:* This shows that there is no universal set (that is, set containing everything).]

Exercise 2 (20 points) A set B is called a *singleton* in case $B = \{x\}$ for some x. Show that there does not exist a set B which contains every singleton. [*Hint:* prove that if there were such a set A, then there would exist a universal set.]

Exercise 3 (3.2 (Chapt. 1) (20 points)) Replace the Axiom of Existence by the following weaker postulate:

> Weak Axiom of Existence Some set exists.
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> Prove the Axiom of Existence using the Weak Axiom of Existence and the Comprehension Schema. [*Hint:* Let A be a set known to exist; consider $\{x \in A \mid x \neq x\}$.]

Exercise 4 (3.6 (Chapt. 1) (20 points)) Show that $\mathcal{P}(X) \subseteq X$ is false for any X. In particular, $\mathcal{P}(X) \neq X$ for any X. This proves that a "set of all sets" does not exist. [*Hint:* Let $Y = \{u \in X \mid u \notin u\}$; $Y \in \mathcal{P}(X)$ but $Y \notin X$.]

Exercise 5 (4.2 (Chapt. 1) (20 points)) Prove:

(i) $A \subseteq B$ if and only if $A \cap B = A$ if and only if $A \cup B = B$ if and only if $A - B = \emptyset$.

(ii) $A \subseteq B \cap C$ if and only if $A \subseteq B$ and $A \subseteq C$.

(iii) $A - B = (A \cup B) - B = A - (A \cap B)$.

(iv) $A = B$ if and only if $A \triangle B = \emptyset$.