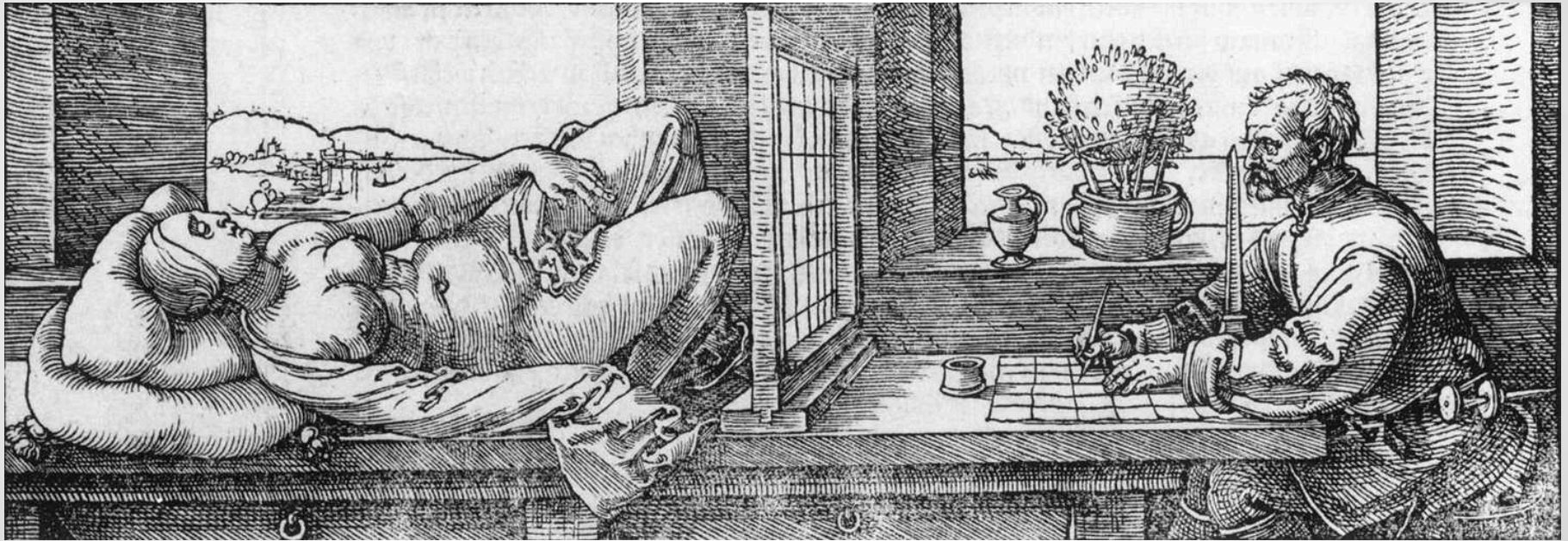


CONCEITOS MATEMÁTICOS

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Do Mundo para Imagem

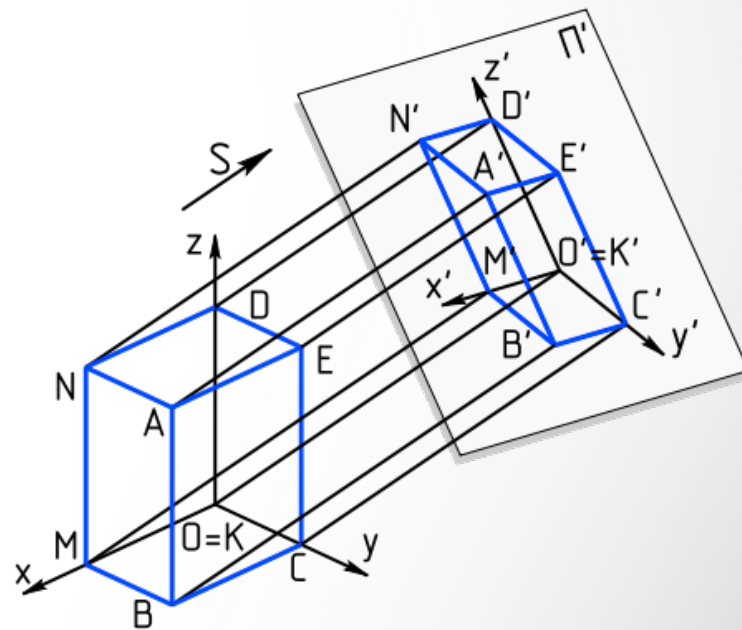
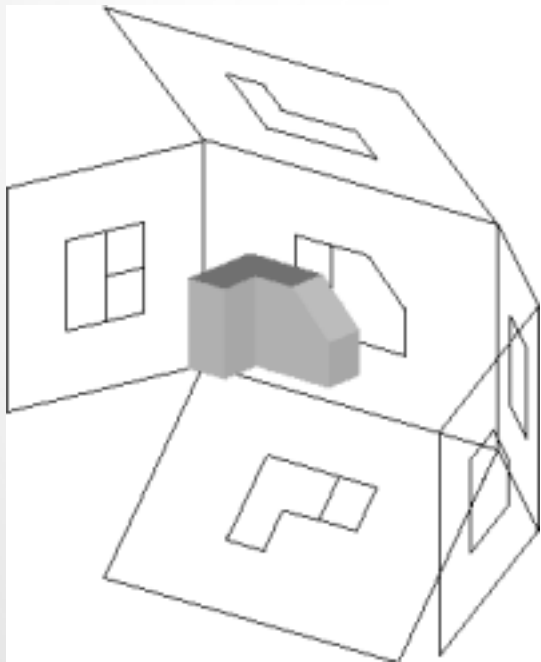


Projeção

- Mapeamento de classes de equivalência
- Representação numa dimensão menor
- Dois tipos

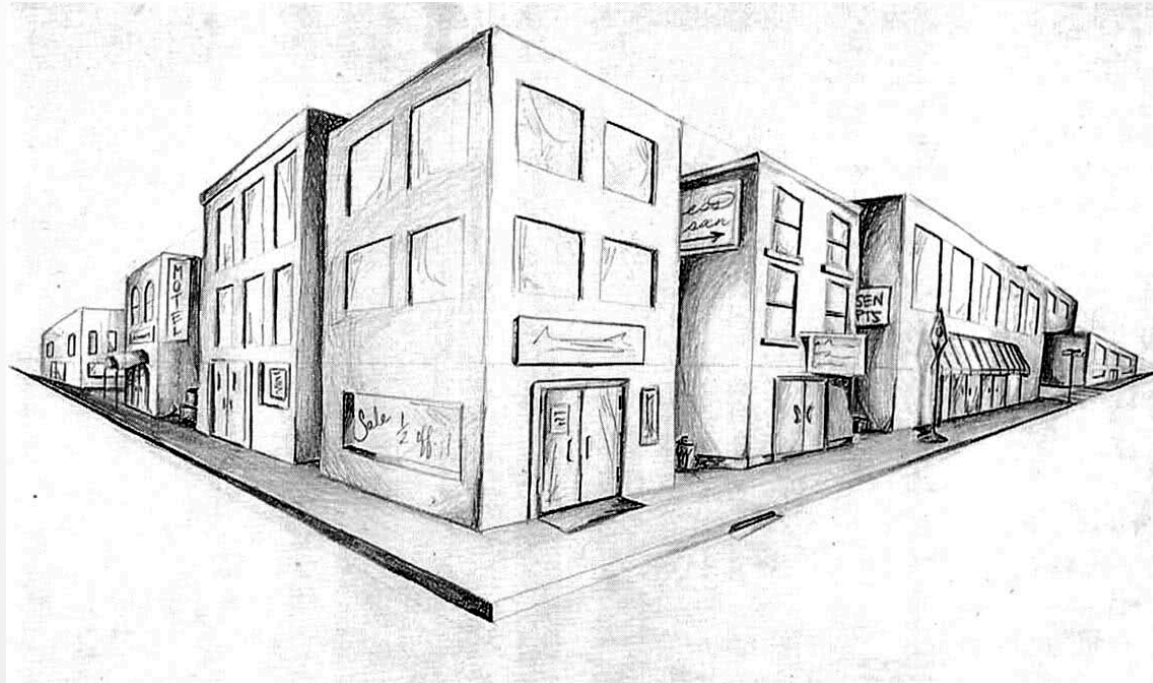
Projeção Ortogonal

- Linhas de projeção ortogonais ao plano
- Transformação afim
- Conveniente para alguns cenários



Projeção Perspectiva

- Linhas de projeção convergente
 - Centro da câmera
- Mais realista
- Pode ter mais de um ponto de fuga

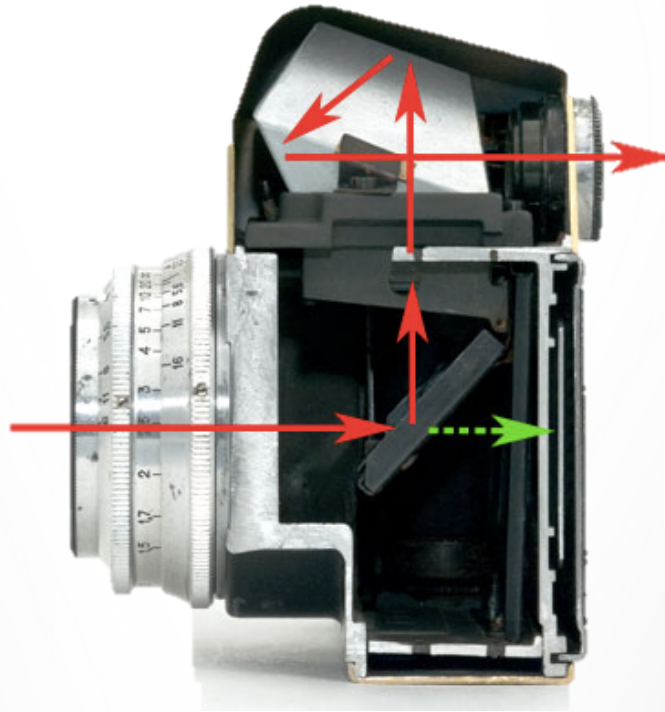


Coordenadas Homogêneas

- Truque matemático para operações não lineares
- Acrescenta uma dimensão
 - 2D $(x, y) \rightarrow (wx, wy, w)$
 - 3D $(x, y, z) \rightarrow (wx, wy, wz, w)$
- Definidos a partir de uma escala

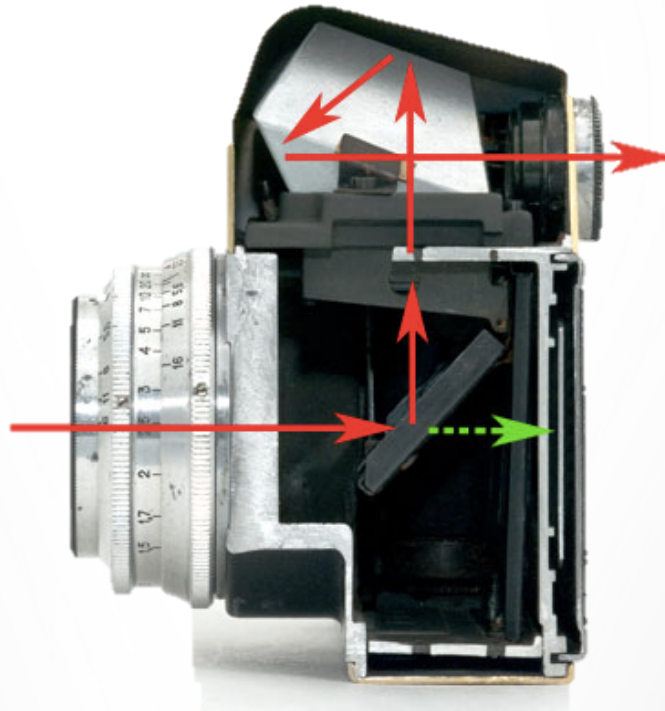
Modelos de Câmera

- Relação matemática entre mundo 3D e o plano de projeção



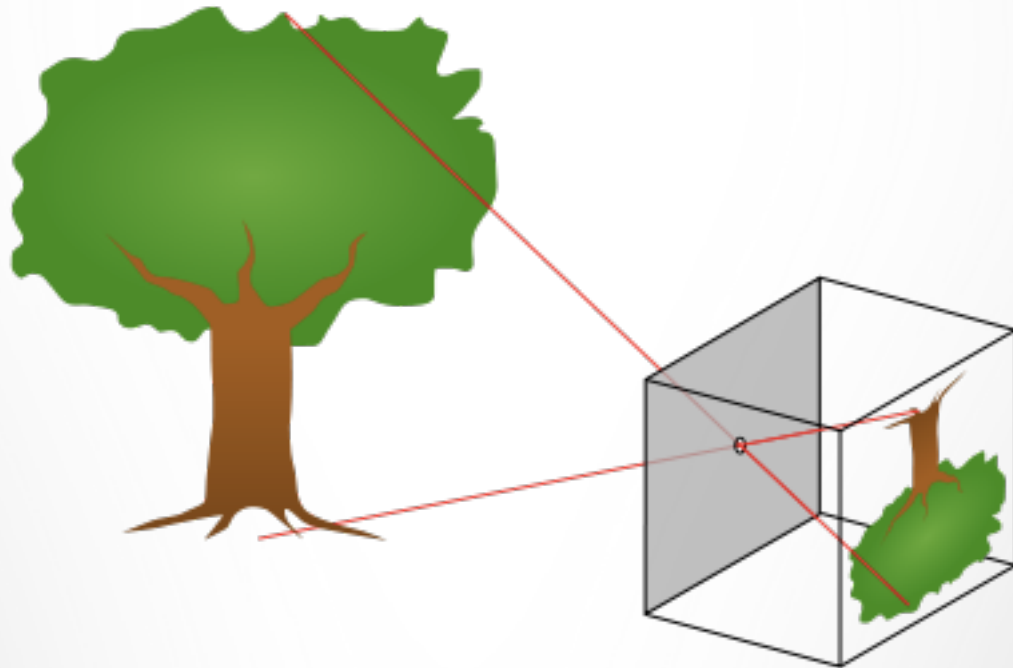
Modelos de Câmera

- Relação matemática entre mundo 3D e o plano de projeção
- Vários modelos
 - Modelo ortográfico
 - Modelo paraperspectivo



Camera Pinhole

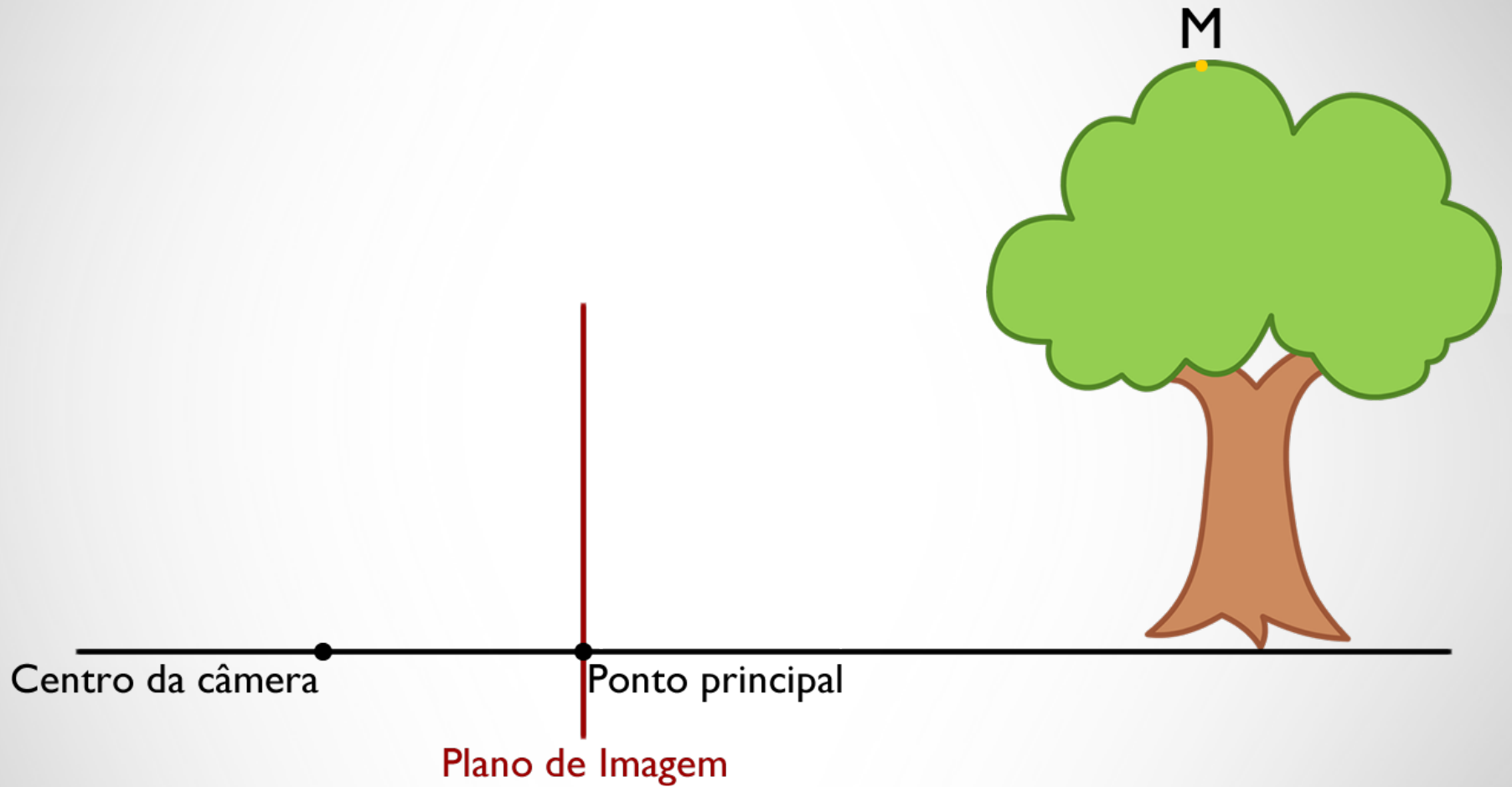
- Modelo simplificado
 - Não considera distorção radial
 - Não considera campo de profundidade
 - Não considera imagens discretas



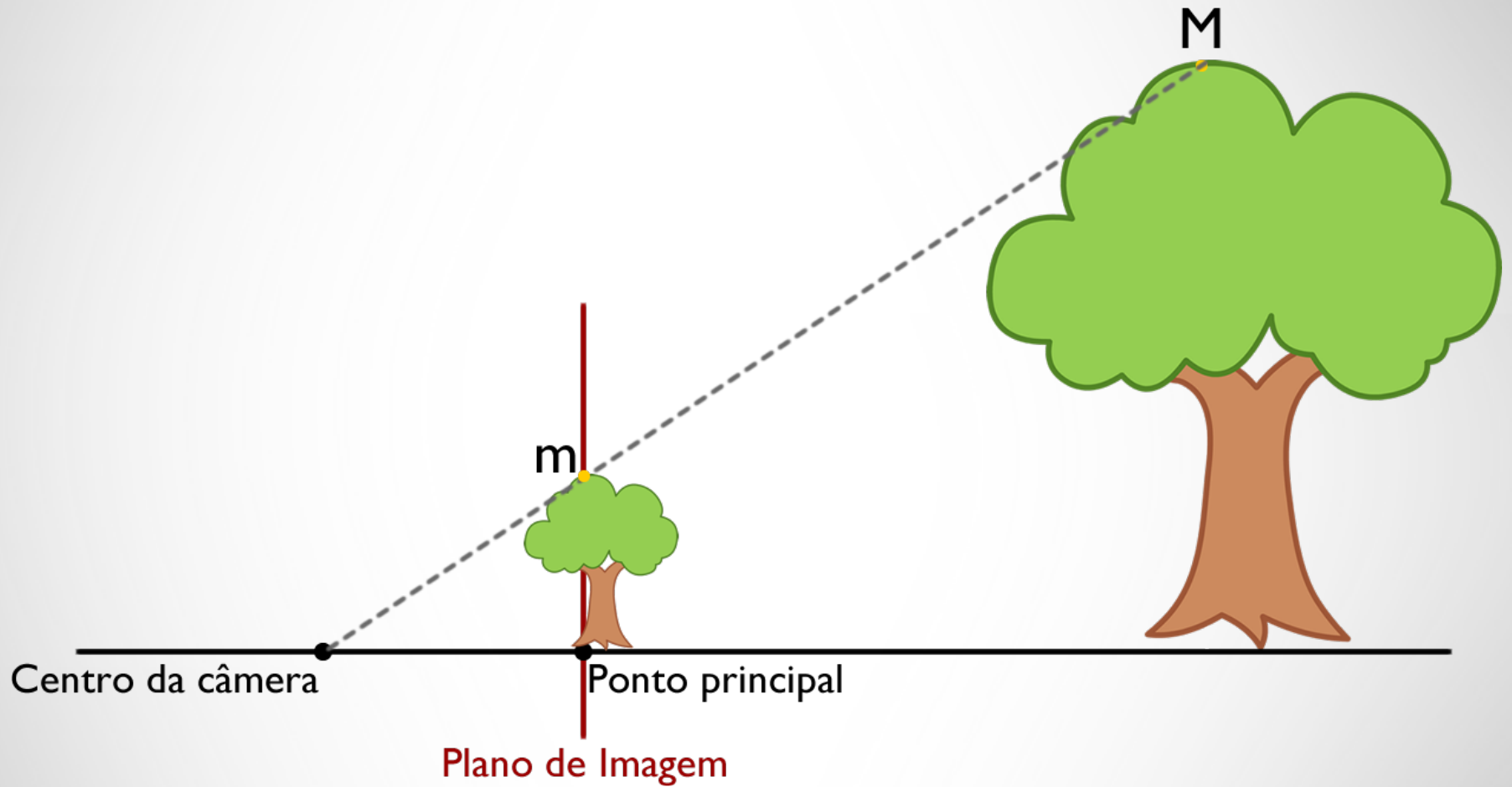
Câmera Pinhole



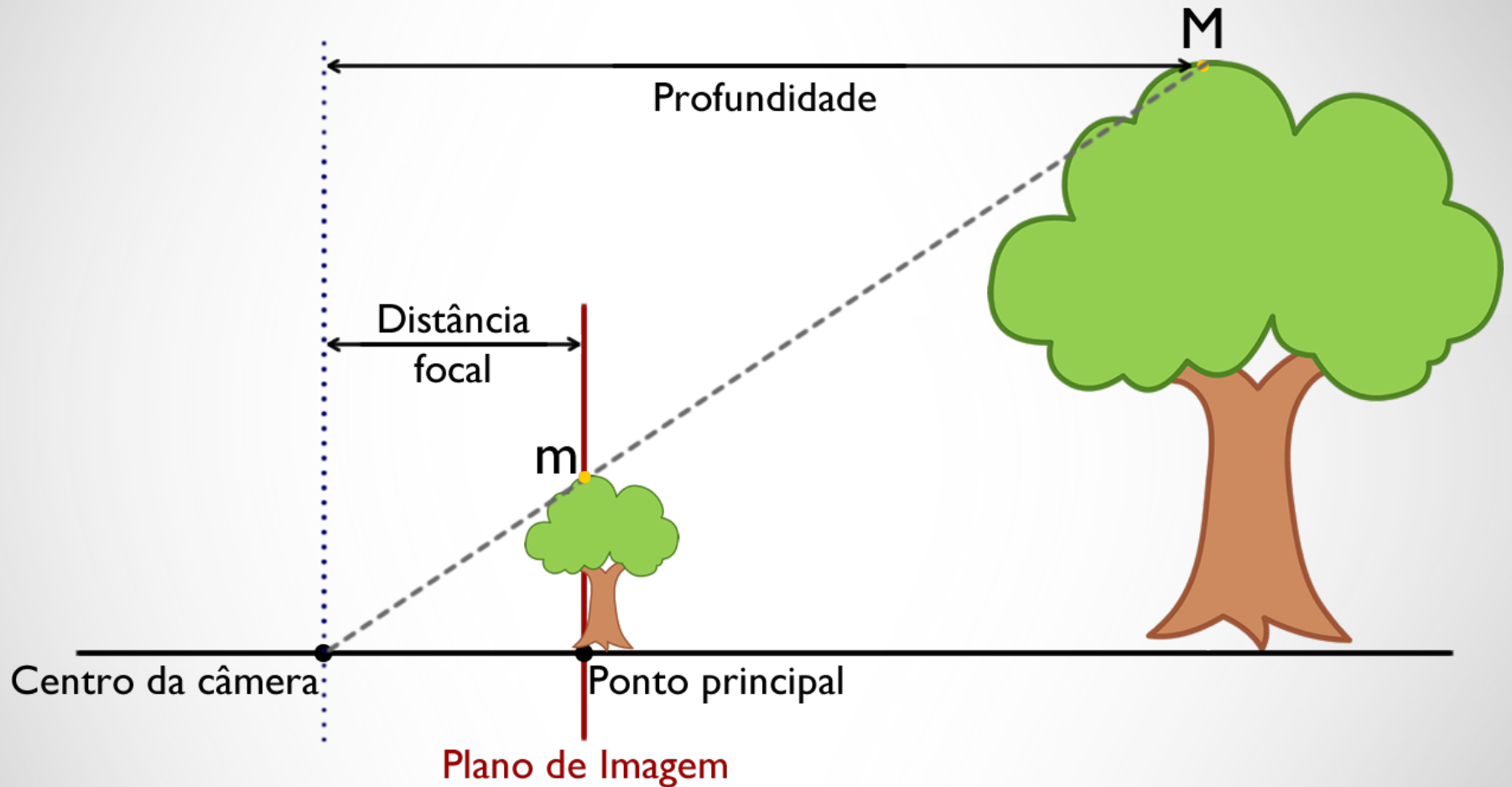
Câmera Pinhole



Câmera Pinhole



Câmera Pinhole

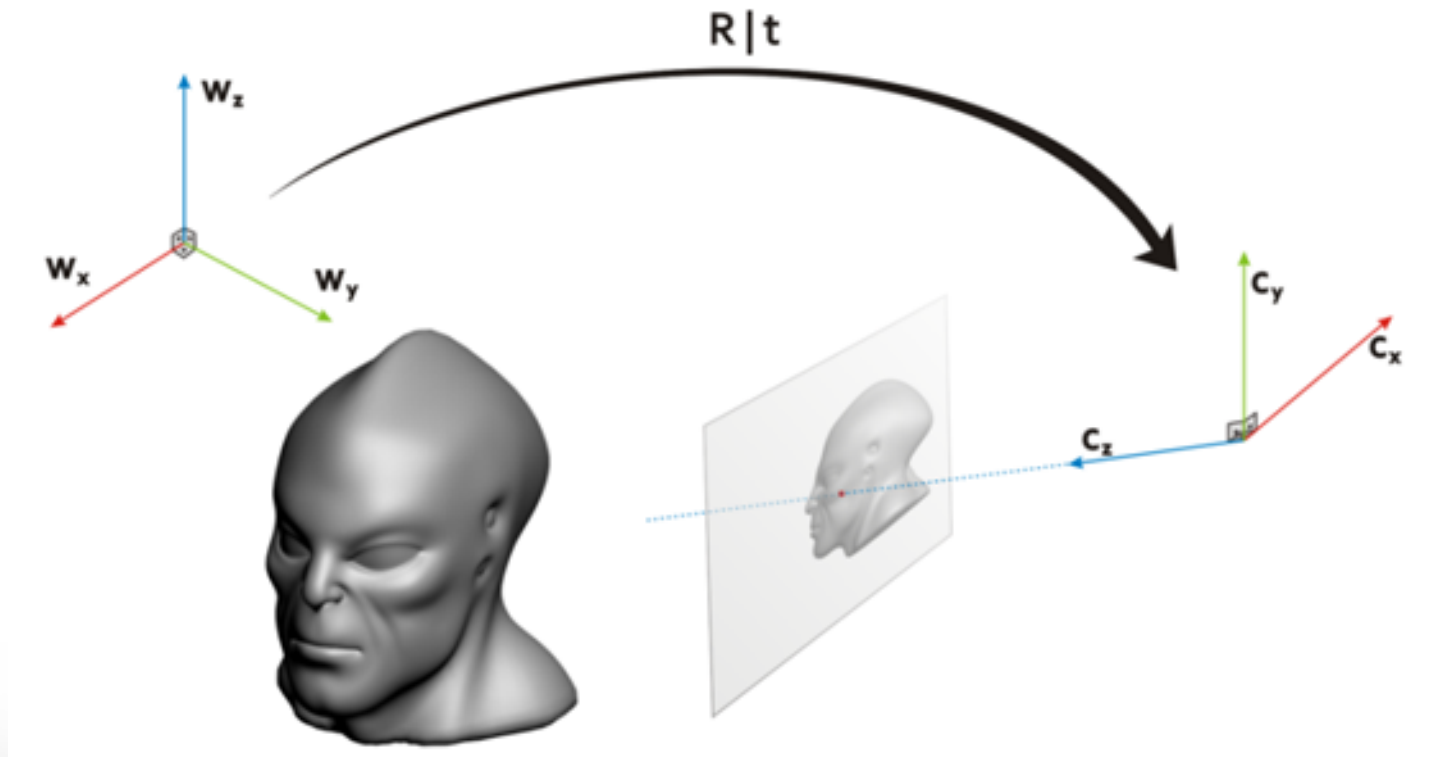


$$\frac{X}{Z} = \frac{x}{f} \therefore x = f \frac{X}{Z}$$

Coordenadas de Câmera

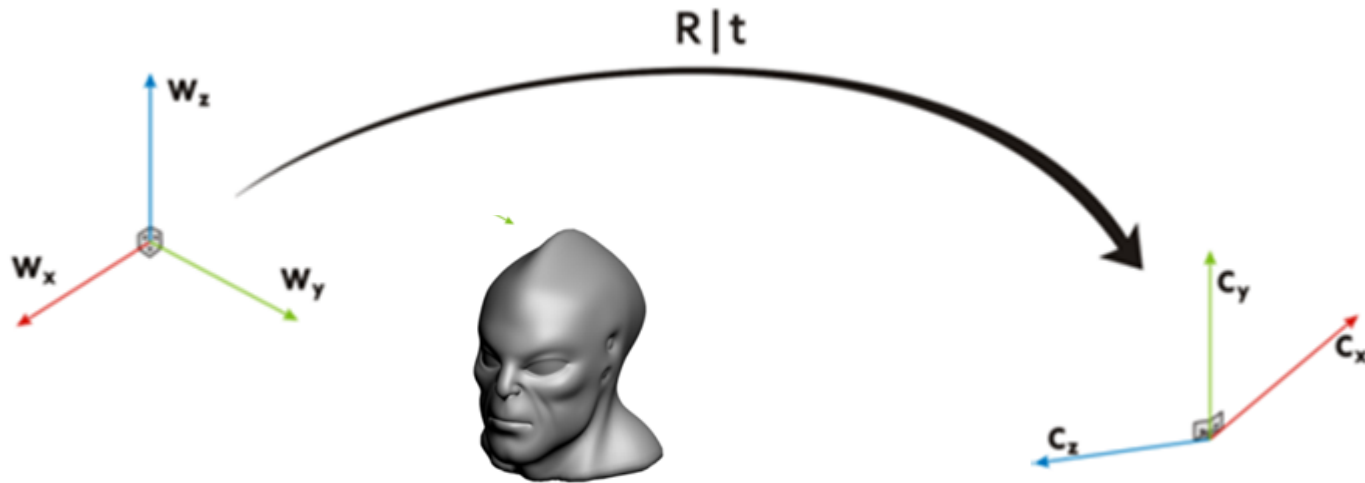
- Coordenadas de mundo \rightarrow coordenadas de câmera

Parâmetros Extrínsecos



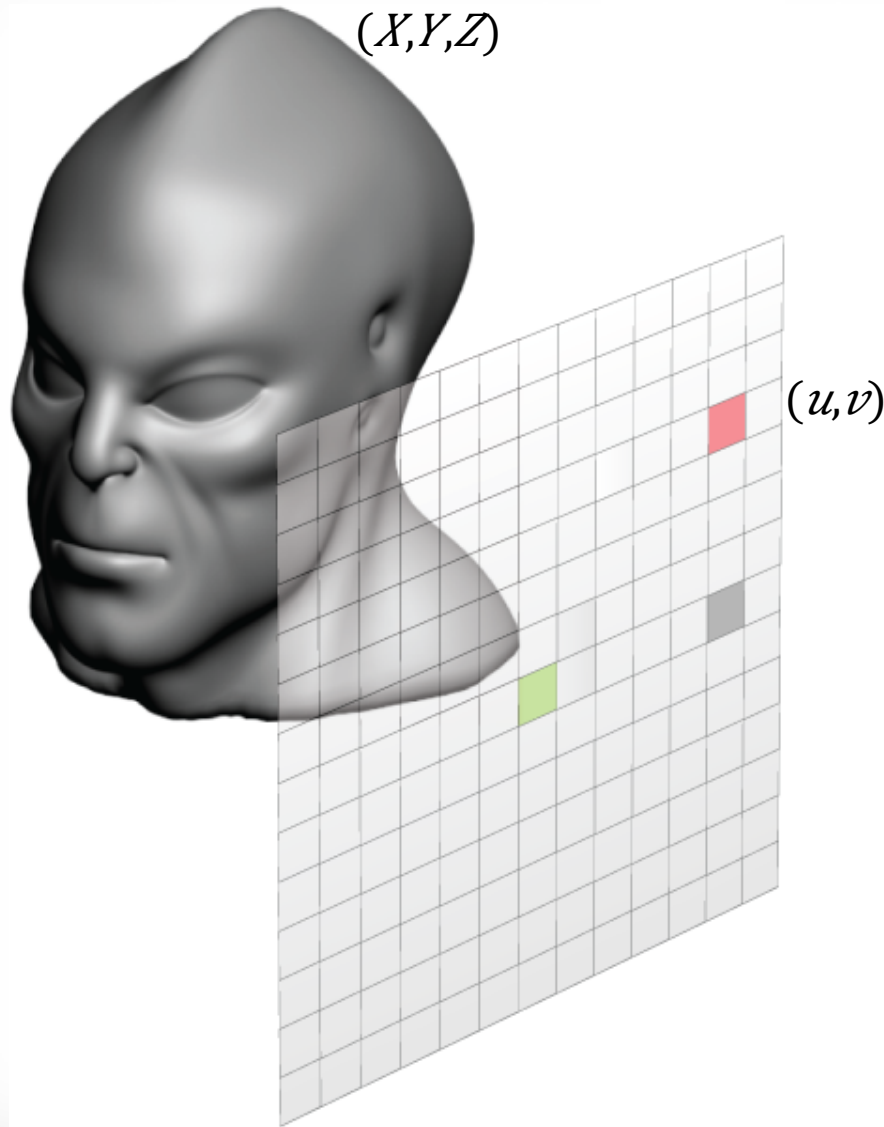
Parâmetros Extrínsecos

- Rotação e Translação



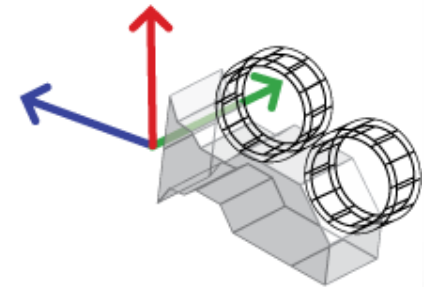
$$\begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_x \\ R_{21} & R_{22} & R_{23} & t_y \\ R_{31} & R_{32} & R_{33} & t_z \end{bmatrix} * \begin{bmatrix} W_x \\ W_y \\ W_z \\ 1 \end{bmatrix}$$

Modelo de Câmera em Perspectiva

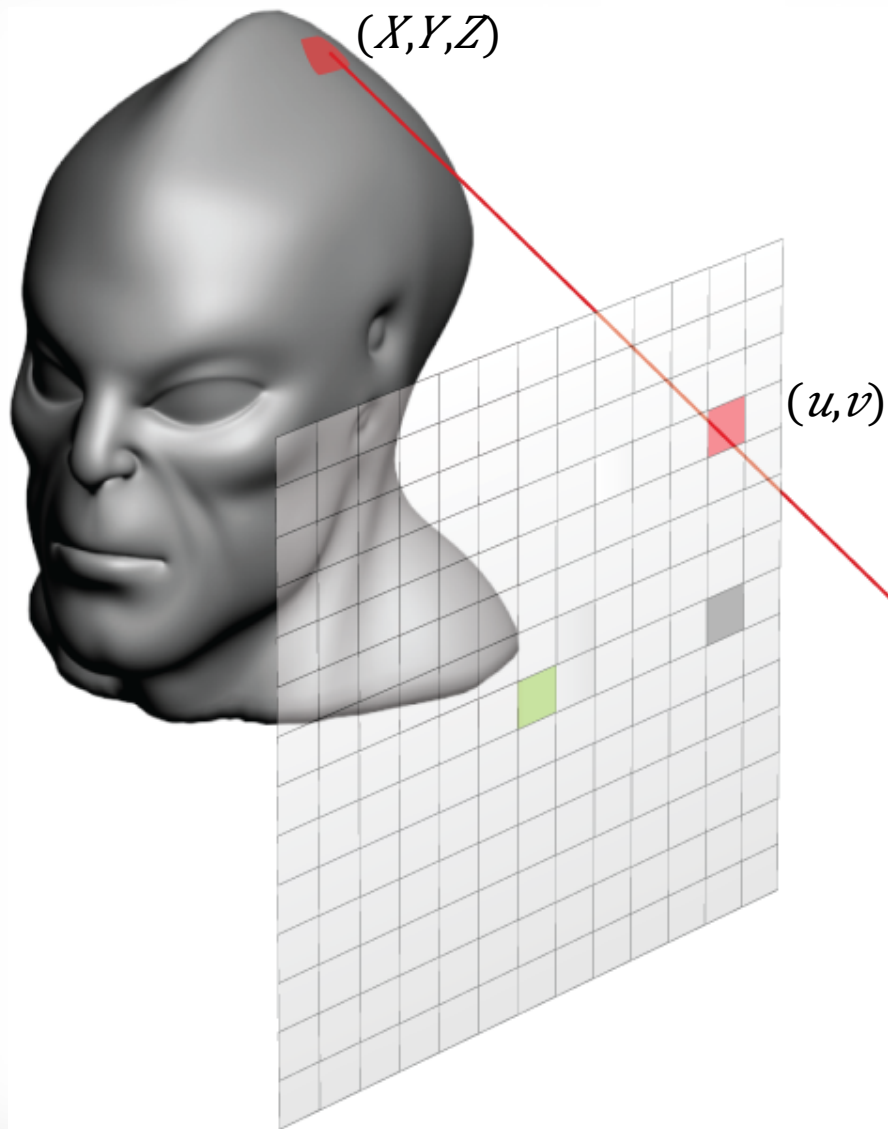


Parâmetros Intrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

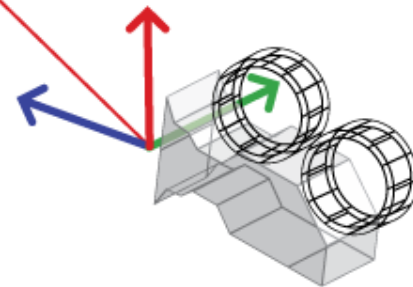


Parâmetros Intrínsecos

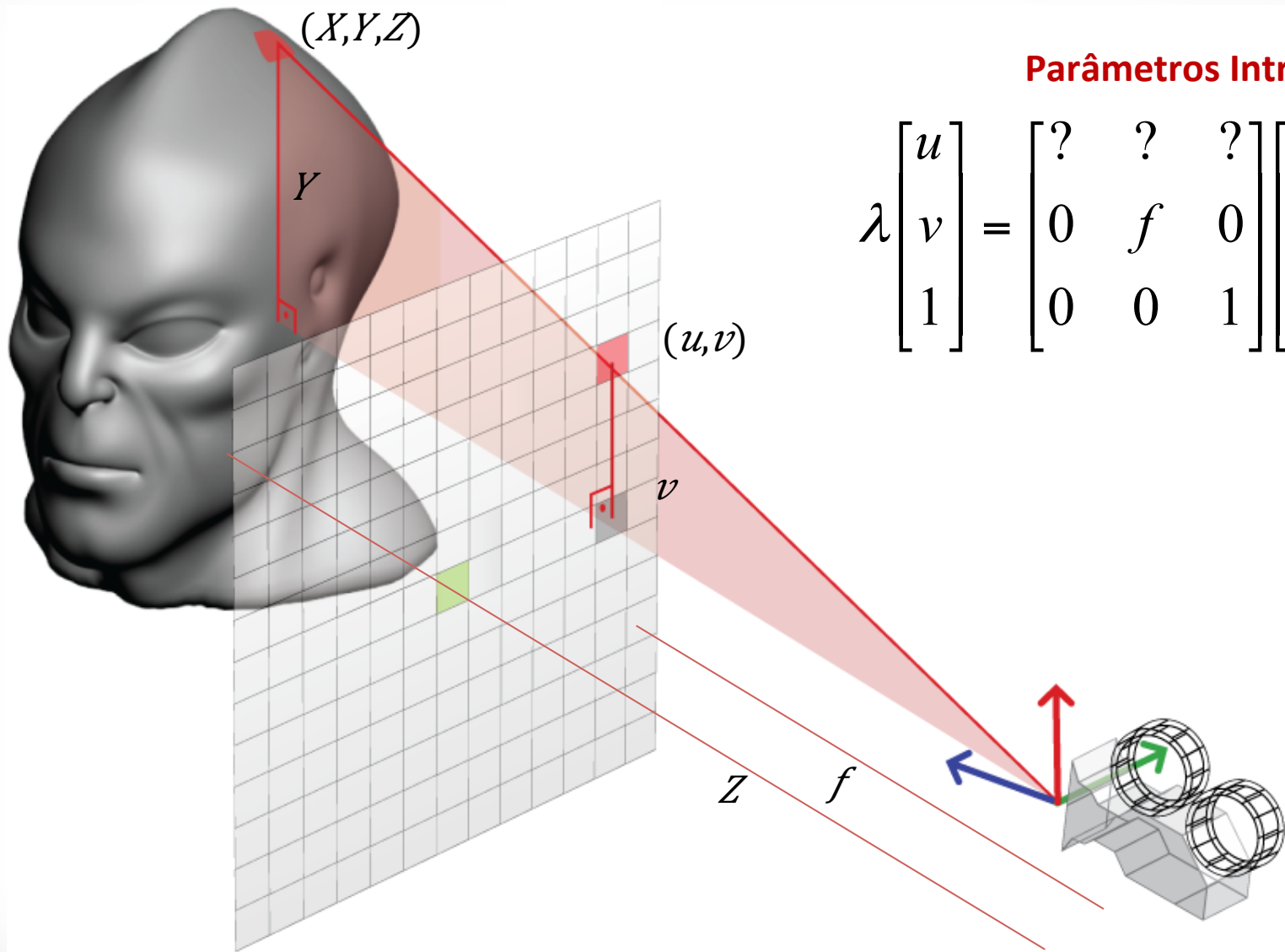


Parâmetros Intrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



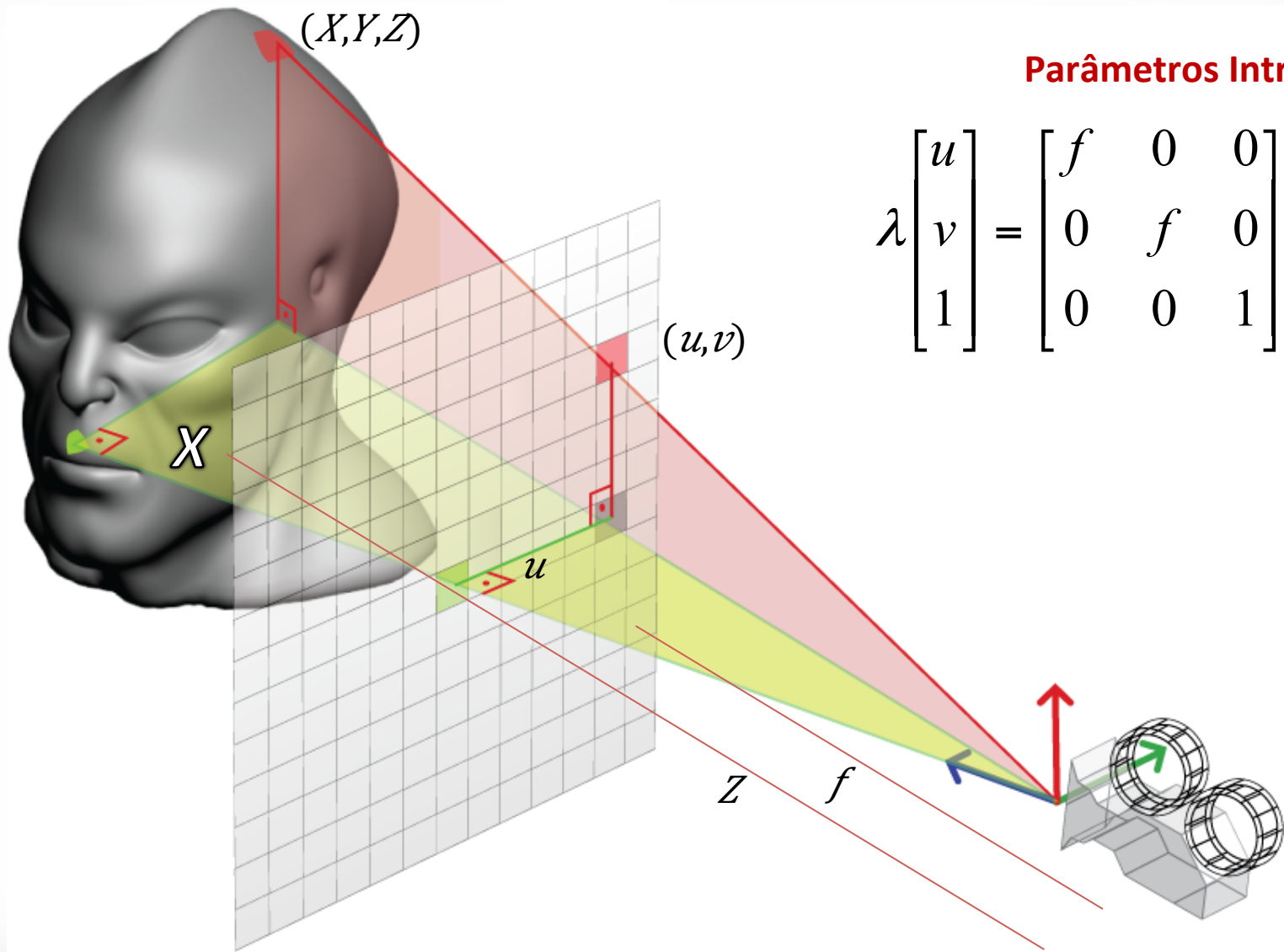
Parâmetros Intrínsecos



Parâmetros Intrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

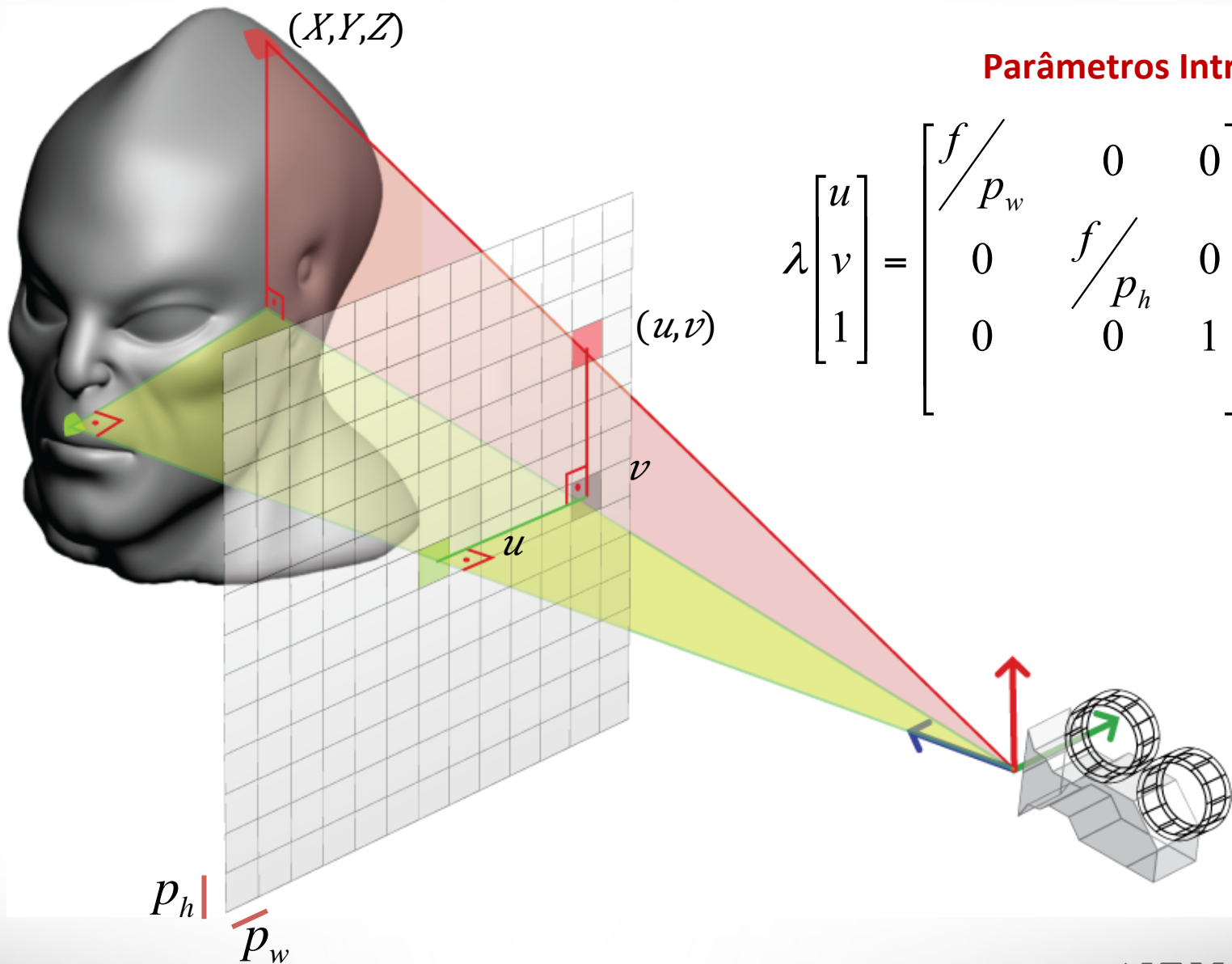
Parâmetros Intrínsecos



Parâmetros Intrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

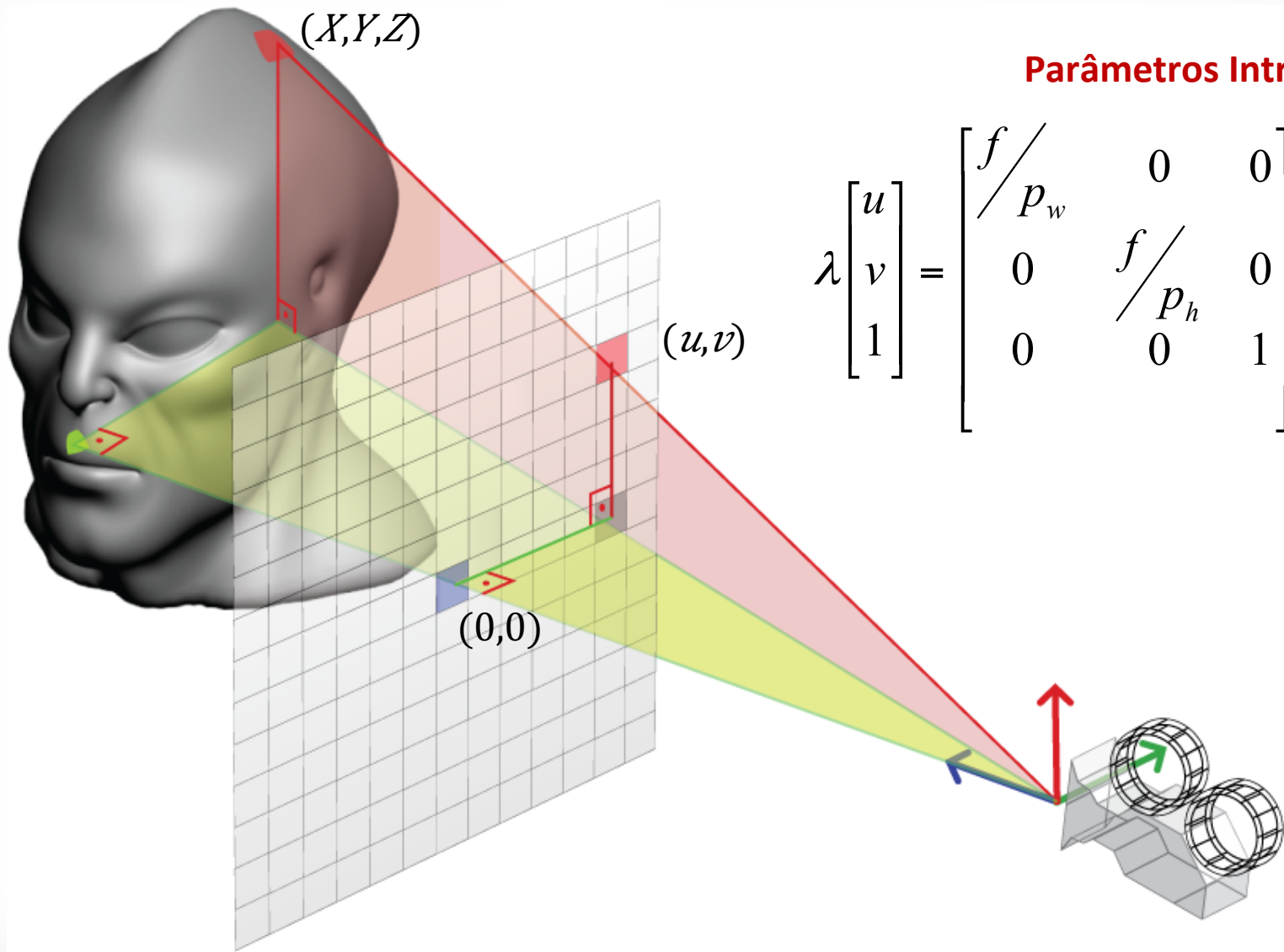
Parâmetros Intrínsecos



Parâmetros Intrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f/p_w & 0 & 0 \\ 0 & f/p_h & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

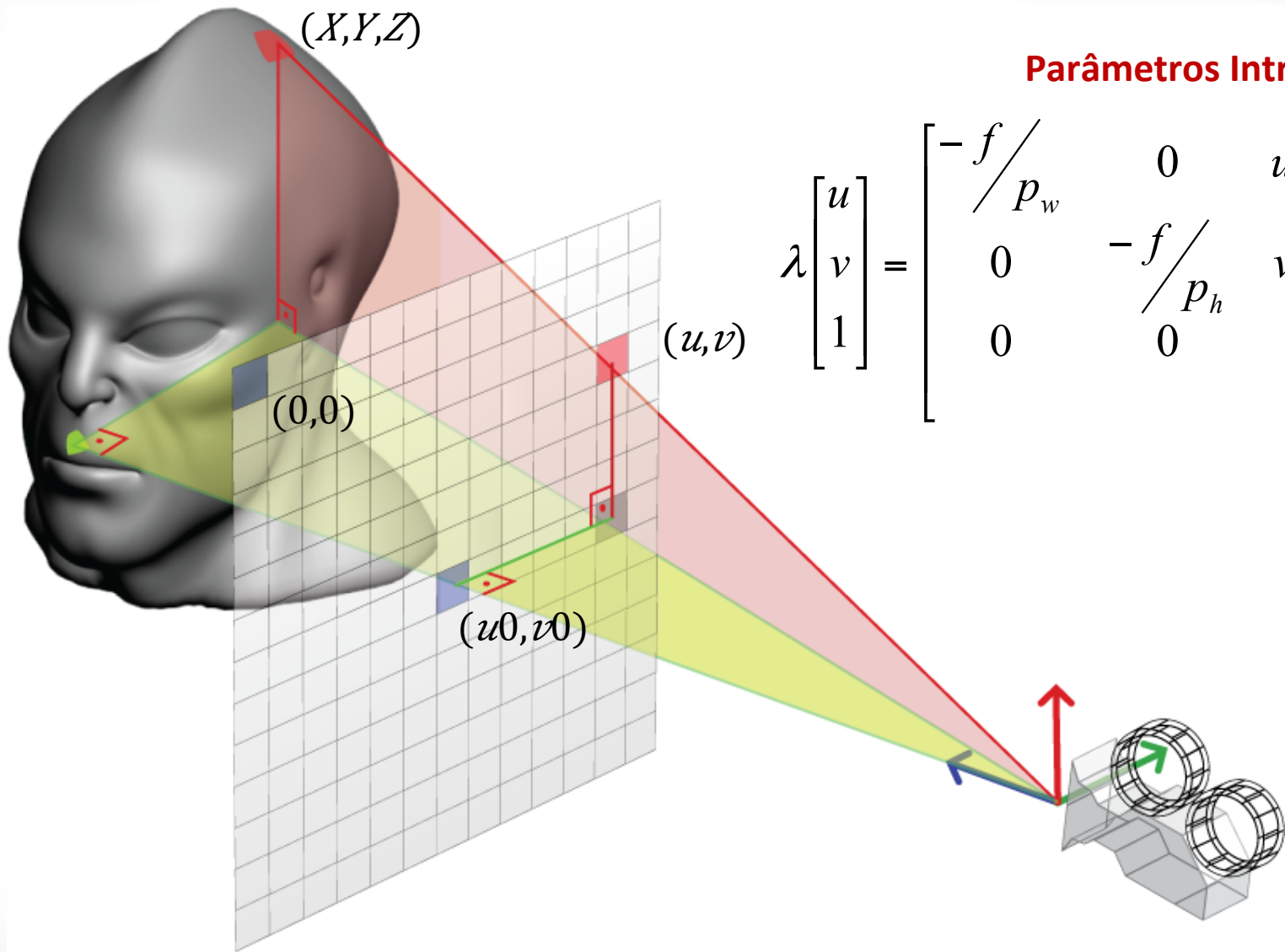
Parâmetros Intrínsecos



Parâmetros Intrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f/p_w & 0 & 0 \\ 0 & f/p_h & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Parâmetros Intrínsecos



Matriz de Projeção

- Composição de parâmetros intrínsecos com extrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f/p_w & 0 & u_0 \\ 0 & f/p_h & v_0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_x \\ R_{21} & R_{22} & R_{23} & t_y \\ R_{31} & R_{32} & R_{33} & t_z \end{bmatrix}}_P * \begin{bmatrix} W_x \\ W_y \\ W_z \\ 1 \end{bmatrix}$$

Matriz de Projeção

- Composição de parâmetros intrínsecos com extrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} W_x \\ W_y \\ W_z \\ 1 \end{bmatrix}$$

Matriz de Projeção

- Composição de parâmetros intrínsecos com extrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f/p_w & 0 & u_0 \\ 0 & f/p_h & v_0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_x \\ R_{21} & R_{22} & R_{23} & t_y \\ R_{31} & R_{32} & R_{33} & t_z \end{bmatrix}}_P * \begin{bmatrix} W_x \\ W_y \\ W_z \\ 1 \end{bmatrix}$$

$$u = \frac{P_{11}W_x + P_{12}W_y + P_{13}W_z + P_{14}}{P_{31}W_x + P_{32}W_y + P_{33}W_z + P_{34}}$$
$$v = \frac{P_{21}W_x + P_{22}W_y + P_{23}W_z + P_{24}}{P_{31}W_x + P_{32}W_y + P_{33}W_z + P_{34}}$$

Matriz de Projeção

- Composição de parâmetros intrínsecos com extrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f/p_w & 0 & u_0 \\ 0 & f/p_h & v_0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_x \\ R_{21} & R_{22} & R_{23} & t_y \\ R_{31} & R_{32} & R_{33} & t_z \end{bmatrix}}_P * \begin{bmatrix} W_x \\ W_y \\ W_z \\ 1 \end{bmatrix}$$

$$P_{11}W_x + P_{12}W_y + P_{13}W_z + P_{14} - P_{31}W_x u - P_{32}W_y u - P_{33}W_z u - P_{34}u = 0$$

$$P_{21}W_x + P_{22}W_y + P_{23}W_z + P_{24} - P_{31}W_x v - P_{32}W_y v - P_{33}W_z v - P_{34}v = 0$$

Matriz de Projeção

- Composição de parâmetros intrínsecos com extrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f/p_w & 0 & u_0 \\ 0 & f/p_h & v_0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_x \\ R_{21} & R_{22} & R_{23} & t_y \\ R_{31} & R_{32} & R_{33} & t_z \end{bmatrix} * \begin{bmatrix} W_x \\ W_y \\ W_z \\ 1 \end{bmatrix}}_P$$

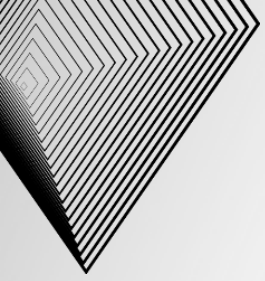
$$\begin{pmatrix} W_x & W_y & W_z & 1 & 0 & 0 & 0 & 0 & -W_x u & -W_y u & -W_z u & -u \\ 0 & 0 & 0 & 0 & W_x & W_y & W_z & 1 & -W_x v & -W_y v & -W_z v & -v \end{pmatrix} \begin{pmatrix} P_{11} \\ \dots \\ P_{34} \end{pmatrix} = \mathbf{0}$$

Matriz de Projeção

- Composição de parâmetros intrínsecos com extrínsecos

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f/p_w & 0 & u_0 \\ 0 & f/p_h & v_0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_x \\ R_{21} & R_{22} & R_{23} & t_y \\ R_{31} & R_{32} & R_{33} & t_z \end{bmatrix}}_P * \begin{bmatrix} W_x \\ W_y \\ W_z \\ 1 \end{bmatrix}$$

$$A * b = \mathbf{0}$$



Conceitos Matemáticos