Matemática Discreta
Lista de Exercícios

Data de entrega: 31/01/03

01. Determine whether each of the following functions is $O(x^2)$.
   a) $f(x) = 17x + 11$
   b) $f(x) = x^2 + 1000$
   c) $f(x) = x \log x$
   d) $f(x) = x^4/2$
   e) $f(x) = 2^x$
   f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

02. Use the definition of the fact that f(x) is O(g(x)) to show that $2^x + 17$ is $O(3^x)$.

03. Show that $(x^3 + 2x)/(2x+1)$ is $O(x^2)$.

04. Find the least integer **n** such that f(x) is $O(x^n)$ for each of the following functions.
   a) $f(x) = 2x^2 + x^3 \log x$
   b) $f(x) = 3x^5 + (\log x)^4$
   c) $f(x) = (x^4+x^2+1)/(x^4+1)$
   d) $f(x) = (x^3+5 \log x)/(x^4+1)$
   e)

05. Show that if f(x) is O(x), then f(x) is $O(x^2)$.

06. For each function, determine whether that function is $\Omega(x)$ and whether it is $\Theta(x)$.
   a) $f(x) = 10$
   b) $f(x) = 3x + 7$
   c) $f(x) = x^2 + x + 1$
   d) $f(x) = 5 \log x$
   e) $f(x) = \lfloor x \rfloor$
   f) $f(x) = \lceil x/2 \rceil$

07. Show that if f(x) and g(x) are functions from the set of real numbers to the set of real numbers, then f(x) is O(g(x)) if and only if g(x) is Ω (f(x)).

08. Show that if a, b, c and d are integers such that a|c and b|d, then ab|cd.

09. Which positive integer less than 12 are relatively prime to 12?

10. Let m be a positive integer. Show that a mod m = b mod m if $a \equiv b \pmod m$.

11. Evaluate the following quantities.
   a) -17 mod 2
   b) 144 mod 7
   c) -101 mod 13
   d) 199 mod 19

12. If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple?

13. Show that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, where a, b, c, d and m are integers with $m \geq 2$, then $a-c \equiv b-d \pmod{m}$.

14. Can you find a formula or rule for the nth term of a sequence related to the prime numbers or prime factorizations so that the initial terms of the sequence have the following values?
    a) 2, 2, 3, 5, 5, 7, 7, 11, 11, 11, 11, 13, 13, …
    b) 0, 1, 2, 2, 3, 3, 4, 4, 4, 4, 5, 5, 6, 6, …
    c) 1, 0, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, …
    d) 1, -1, -1, 0, -1, 1, -1, 0, 0, 1, -1, 0, -1, 1, 1, …
    e) 1, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, …
    f) 4, 9, 25, 49, 121, 169, 289, 361, 529, 841, 961, 1369, …