(b) Prove that if we assume that the deleted edge e belongs to a cycle that is a subgraph of G, then the remaining graph is connected.

7.13 Let G be a graph and let u and v be two nodes of G.

(a) Prove that if there is a walk in G from u to v, then G contains a path connecting u and v.

(b) Use part (a) to give another proof of the fact that if G contains a path connecting a and b, and also a path connecting b and c, then it contains a path connecting a and c.

7.14 Let G be a graph, and let $H_1 = (V_1, E_1)$ and $H_2 = (V_2, E_2)$ be two subgraphs of G that are connected. Assume that H_1 and H_2 have at least one node in common. Form their union, i.e., the subgraph $H = (V', E')$, where $V' = V_1 \cup V_2$ and $E' = E_1 \cup E_2$. Prove that H is connected.

7.15 Determine the connected components of the graphs constructed in exercise 7.4.

7.16 Prove that no edge of G can connect nodes in different connected components.

7.17 Prove that a node v is a node of the connected component of G containing node u if and only if g contains a path connecting u to v.

7.18 Prove that a graph with n nodes and more than $\binom{n-1}{2}$ edges is always connected.